Renewal for the project team (EPC) for the period (2022–2025)

TRIPOP

Modelling, Simulation and Control of Nonsmooth Dynamical Systems with application to natural environmental risks modelling in mountains

Modélisation, simulation et commande des systèmes dynamiques non-lisses avec une application pour le risque naturel environmental en montagne

January 25, 2023

- **Domain** : Applied Mathematics, Computation and Simulation
- **Theme** : Optimization and control of dynamic systems
- **Associated institutions "établissements de rattachement"** : Inria and Grenoble-INP, CNRS through the Laboratoire Jean Kuntzman (LJK UMR CNRS 5524, Dir. Jean Guillaume Dumas).

Abstract

This document presents the renewal of the joint research team, TRIPOP, between INRIA Grenoble Rhône–Alpes, Grenoble INP and CNRS, part of the Laboratoire Jean Kuntzmann (LJK UMR 5224) after its first period of evaluation (2018–2021). The team is mainly concerned with the modelling, the mathematical analysis, the simulation and the control of nonsmooth dynamical systems, with a strong application to modelling natural environmental risks in mountains. Nonsmooth dynamics concerns the study of the time evolution of systems that are not smooth in the mathematical sense, *i.e.* systems that are characterized by a lack of differentiability, either of the mappings in their formulations, or of their solutions with respect to time. In mechanics, the main instances of nonsmooth dynamical systems are multibody systems with Signorini unilateral contact, set-valued (Coulomb-like) friction and impacts. In electronics, examples are found in switched electrical circuits with ideal components (diodes, switches, transistors). In control, nonsmooth systems arise in the sliding mode control theory and in optimal control. Many examples can also be found in cyber-physical systems (hybrid systems), in transportation sciences, in mathematical biology or in finance. For the next four years, the team is organized along two research axes: 1) nonsmooth simulation and numerical modelling for natural gravitational risks in mountains and 2) modelling, simulation and control of nonsmooth dynamical systems. The idea of this restructuring is to put forward a strong application axis for which there is a strong academic research dynamic in the Grenoble region and a network of socio-economic actors very interested in an industrial transfer of digital science methods on these subjects. The second axis takes up the main themes of the former axes of the TRIPOP project by updating them after the first four years.

1 TRIPOP team

This section describes the composition of the team.

1.1 Team Members / *Membres de l'équipe*

The members of the team are:

- Research Scientists
  - Vincent Acary Inria, DR2 (LJK, HdR, Team Leader)
  - Franck Bourrier, INRAE, CRCN (LJK, HdR)
  - Bernard Brogliato, Inria, DR1 (LJK, HdR)
  - Olivier Goury, Inria, CRCN (LJK)
  - Felix Miranda Villatoro, Inria, ISFP (LJK)
  - Arnaud Tonnelier, Inria, CRN (LJK, HdR)
- Faculty members
  - Paul Armand, Univ Limoges Professor (XLIM, HdR)
  - Guillaume James, Grenoble INP, Professor (LJK, HdR)
- Post-doctoral fellows
  - Nicholas Collins-Craft Inria, granted by MSCA Fellowship.
- PhD students
  - Louis Guillet granted by INRIA, AEx GRANIER
  - Hoang Minh Nguyen granted by INRIA,
  - Quand Hung Pham granted by Grenoble INP ED EEATS,
  - Aya Younes granted by UGA ED EEATS,
- Technical staff
  - Maurice Brémond, Inria, IR1 (LJK, SED, 40%)
  - Samuel Heidmann, Inria, IR2 (LJK, SED, 90%)
  - Franck Pérignon, CNRS, IR1 (LJK, 20%)
- Administrative assistant:
  - Diane Courtiol, Inria.
- External collaborator
  - Christophe Prieur, CNRS, DR1 (Gipsa-Lab UMR 5216, HdR)

2 General scope and motivations

Nonsmooth dynamics concerns the study of the time evolution of systems that are not smooth in the mathematical sense, *i.e.*, systems that are characterized by a lack of differentiability, either of the mappings in their formulations, or of their solutions with respect to time. The class of nonsmooth dynamical systems recovers a large variety of dynamical systems that arise in many applications. The term "nonsmooth", like the term "nonlinear", does not precisely define the scope of the systems we are interested in but, and most importantly, they are characterized by the mathematical and numerical properties that they share. To give more insight into nonsmooth dynamical systems, we give in the following a very brief introduction of their salient features. For more details, we refer to Acary and Brogliato (2008), Bernardo, Budd, Champneys, and Kowalczyk (2008), Brogliato (2016), Filippov (1988), Goeleven (2017), and Leine and Wouw (2008).

As we have indicated there are many applications to the methods of nonsmooth dynamics. We have chosen a strong particular application for this technique of nonsmooth dynamics which is that of natural gravity risk in the mountains. The choice of this application is particularly motivated by global climate change which has increased the number of rockfall and landslide events very significantly in recent decades. Especially, the effects of melting permafrost, increased rainfall and rapid temperature changes means that alpine regions are particularly at risk (Keiler, Knight, and Harrison, 2010; Rist, 2007). Another important interest is the strong academic research dynamics in the Grenoble region and a network of socio-economic actors very interested in an industrial transfer of digital science methods on these subjects. The team will conduct research on the mechanical modelling and simulation of natural hazards in mountains (floods and debris flows, block falls, glacial hazards), bringing new software development in a high performance computing (HPC) framework.

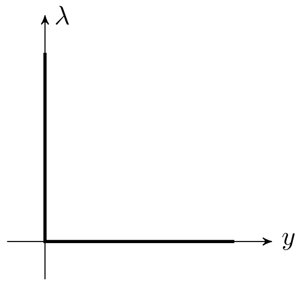


Figure 1: Complementarity condition $0 \leq y \perp \lambda \geq 0$.

2.1 A flavor of nonsmooth dynamical systems

As a first illustration, let us consider a linear finite-dimensional system described by its state $x(t) \in \mathbb{R}^n$ over a time-interval $t \in [0, T]$:

$$\dot{x}(t) = Ax(t) + a, \quad A \in \mathbb{R}^{n\times n},\, a \in \mathbb{R}^n, \tag{1}$$

subjected to a set of m inequality (unilateral) constraints:

$$y(t) = Cx(t) + c \geq 0, \qquad C \in \mathbb{R}^{m\times n},\, c \in \mathbb{R}^m. \tag{2}$$

If the constraints are physical constraints, a standard modelling approach is to augment the dynamics in (1) by an input vector $\lambda(t) \in \mathbb{R}^m$ that plays the role of a Lagrange multiplier vector. The multiplier restricts the trajectory of the system in order to respect the constraints. Furthermore, as in the continuous optimization theory, the multiplier must be signed and must vanish if the constraint is not active. This is usually formulated as a complementarity condition:

$$0 \leq y(t) \perp \lambda(t) \geq 0, \tag{3}$$

which models the one-sided effect of the inequality constraints. The notation $y \geq 0$ holds component–wise and $y \perp \lambda$ means $y^T\lambda = 0$. All together we end up with a Linear Complementarity System (LCS) of the form,

$$\begin{cases} \dot{x}(t) = Ax(t) + a + B\lambda(t) \\ y(t) = Cx(t) + c \\ 0 \leq y(t) \perp \lambda(t) \geq 0 \end{cases} \tag{4}$$

where $B \in \mathbb{R}^{n\times m}$ is the matrix that models the input generated by the constraints. In a more general way, the constraints may also involve the Lagrange multiplier,

$$y(t) = Cx(t) + c + D\lambda(t) \geq 0, \qquad D \in \mathbb{R}^{m\times m}, \tag{5}$$

leading to a general definition of LCS as

$$\begin{cases} \dot{x}(t) = A\,x(t) + a + B\,\lambda(t) \\ y(t) = C\,x(t) + c + D\,\lambda(t) \\ 0 \leq y(t) \perp \lambda(t) \geq 0. \end{cases} \tag{6}$$

The complementarity condition, illustrated in Figure 1 is the archetype of a nonsmooth graph that we extensively use in nonsmooth dynamics. The mapping $y \mapsto \lambda$ is a multi-valued (set-valued) mapping, that is nonsmooth at the origin. It has many interesting mathematical properties and reformulations that come mainly from convex analysis and variational inequality theory. Let us introduce the indicator function of $\mathbb{R}_+$ as

$$\Psi_{\mathbb{R}_+}(x) = \begin{cases} 0 \text{ if } x \geq 0, \\ +\infty \text{ if } x < 0. \end{cases} \tag{7}$$

This function is convex, proper and can be sub-differentiated (Hiriart-Urruty and Lemaréchal, 2001). The definition of the subdifferential of a convex function $f : \mathbb{R}^m \to \mathbb{R}$ is defined as:

$$\partial f(x) = \{x^\star \in \mathbb{R}^m \mid f(z) \geq f(x) + (z-x)^\top x^\star, \forall z\}. \tag{8}$$

A basic result of convex analysis is

$$0 \le y \perp \lambda \ge 0 \Longleftrightarrow -\lambda \in \partial\Psi_{I\!R_+}(y) \tag{9}$$

that gives a first functional meaning to the set-valued mapping $y \mapsto \lambda$. Another interpretation of $\partial\Psi_{I\!R_+}$ is based on the normal cone to a closed and nonempty convex set C:

$$N_C(x) = \{v \in I\!R^m | v^\top(z-x) \le 0 \text{ for all } z \in C\}. \tag{10}$$

It is easy to check that $\partial\Psi_{I\!R_+}(x) = N_{I\!R_+}(x)$ and it follows that

$$0 \le y \perp \lambda \ge 0 \Longleftrightarrow -\lambda \in N_{I\!R_+}(y). \tag{11}$$

Finally, the definition of the normal cone yields a variational inequality:

$$0 \le y \perp \lambda \ge 0 \Longleftrightarrow \lambda^\top(y-z) \le 0, \forall z \ge 0. \tag{12}$$

The relations (11) and (12) allow one to formulate the complementarity system with $D = 0$ as a differential inclusion based on a normal cone (see (15)) or as a differential variational inequality. By extending the definition to other types of convex functions, possibly nonsmooth, and using more general variational inequalities, the same framework applies to the nonsmooth laws depicted in Figure 2 that includes the case of piecewise smooth systems.

The mathematical concept of solutions depends strongly on the nature of the matrix quadruplet (A, B, C, D) in (6). If D is a positive definite matrix (or a P-matrix), the Linear Complementarity problem

$$0 \le Cx + c + D\lambda \perp \lambda \ge 0, \tag{13}$$

admits a unique solution $\lambda(x)$ which is a Lipschitz continuous mapping. It follows that the Ordinary Differential Equation (ODE)

$$\dot{x}(t) = Ax(t) + a + B\lambda(x(t)), \tag{14}$$

is a standard ODE with a Lipschitz right-hand side with a C^1 solution for the initial value problem. If $D = 0$, the system can be written as a differential inclusion in a normal cone as

$$-\dot{x}(t) + Ax(t) + a \in BN_{I\!R_+}(Cx(t)), \tag{15}$$

that admits a solution that is absolutely continuous if CB is a definite positive matrix and the initial condition satisfies the constraints. The time derivative $\dot{x}(t)$ and the multiplier $\lambda(t)$ may have jumps and are generally considered as functions of bounded variations. If $CB = 0$, the order of nonsmoothness increases and the Lagrange multiplier may contain Dirac atoms and must be considered as a measure. Higher–order index, or higher relative degree systems yield solutions in terms of distributions and derivatives of distributions (Acary, Brogliato, and Goeleven, 2008).

A lot of variants can be derived from the basic form of linear complementarity systems, by changing the form of the dynamics including nonlinear terms or by changing the complementarity relation by other multivalued maps. In particular the nonnegative orthant may be replaced by any convex closed cone $K \subset I\!R^m$ leading to complementarity over cones

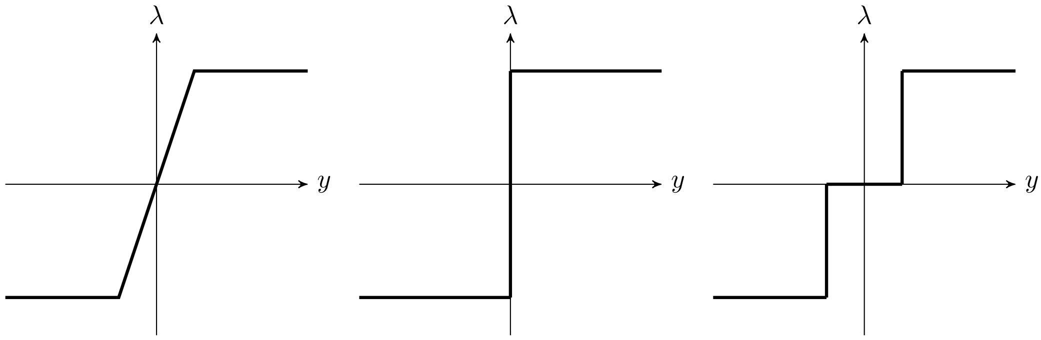
$$K^\star \ni y \perp \lambda \in K, \tag{16}$$

where $K^\star$ its dual cone given by

$$K^\star = \{x \in I\!R^m \mid x^\top y \ge 0 \text{ for all } y \in K\}. \tag{17}$$

In Figure 2, we illustrate some other basic maps that can be used for defining the relation between λ and y. The saturation map, depicted in Figure 2(a) is a single valued continuous function which is an archetype of a piece-wise smooth map. In Figure 2(b), the relay multi-function is illustrated. If the upper and the lower limits of λ are respectively equal to 1 and -1, we obtain the multivalued sign function defined as

$$\mathrm{Sgn}(y) = \begin{cases} 1, & y > 0 \\ [-1, 1], & y = 0 \\ -1, & y < 0. \end{cases} \tag{18}$$



(a) Saturation (b) Relay (c) Relay with dead zone

Figure 2: Examples of multivalued piecewise linear models

Using again convex analysis, the multivalued sign function may be formulated as an inclusion into a normal cone as

$$\lambda \in \mathrm{Sgn}(y) \Longleftrightarrow y \in N_{[-1,1]}(\lambda). \tag{19}$$

More generally, any system of the type,

$$\begin{cases} \dot{x}(t) = Ax(t) + a + B\lambda(t) \\ y(t) = Cx(t) + a \\ -\lambda(t) \in \mathrm{Sgn}(y(t)), \end{cases} \tag{20}$$

can reformulated in terms of the following set-valued system

$$\begin{cases} \dot{x}(t) = Ax(t) + a + B\lambda(t) \\ y(t) = Cx(t) + c \\ -y(t) \in N_{[-1,1]^m}(\lambda(t)). \end{cases} \tag{21}$$

The system (21) appears in a lot of applications; among them, we can cite the sliding mode control, electrical circuits with relay and Zener diodes (Acary, Bonnefon, and Brogliato, 2011), or mechanical systems with friction (Acary and Brogliato, 2008).

Though this class of systems seems to be rather specific, it includes as well more general dynamical systems such as piecewise smooth systems and discontinuous ordinary differential equations. Indeed, the system (20) for scalars y and λ can be viewed as a discontinuous differential equation:

$$\dot{x}(t) = \left\{ \begin{array}{lll} Ax + a + B & \text{if} & Cx + c > 0 \\ Ax + a - B & \text{if} & Cx + c < 0. \end{array} \right. \tag{22}$$

One of the most well-known mathematical frameworks to deal with such systems is the Filippov theory (Filippov, 1988) that embeds the discontinuous differential equations into a differential inclusion. In the case of a single discontinuity surface given in our example by $S = \{x \mid Cx + c = 0\}$, the Filippov differential inclusion based on the convex hull of the vector fields in the neighborhood of S is equivalent to the use of the multivalued sign function in (20). Conversely, as it has been shown in Acary, Jong, and Brogliato (2013), a piecewise smooth system can be formulated as a nonsmooth system based on products of multivalued sign functions.

2.2 Nonsmooth Dynamical systems in the large

Generally, the nonsmooth dynamical systems we propose to study mainly concern systems that possess the following features:

(i) A nonsmooth formulation of the constitutive/behavioral laws that define the system. Examples of nonsmooth formulations are piecewise smooth functions, multi–valued functions, inequality constraints, yielding various definitions of dynamical systems such as piecewise smooth systems, discontinuous ordinary differential equations, complementarity systems, projected dynamical systems, evolution or differential variational inequalities and differential inclusions (into normal cones). Fundamental mathematical tools come from convex analysis (Hiriart-Urruty and Lemaréchal, 1993, 2001; Rockafellar, 1970), complementarity theory (Cottle, Pang, and Stone, 1992), and variational inequalities theory (Facchinei and Pang, 2003).

(ii) A concept of solutions that does not require continuously differentiable functions of time. For instance, absolutely continuous, Lipschitz continuous functions or functions of local bounded variation are the basis for solution concepts. Measures or distributions are also solutions of interest for differential inclusions or evolution variational inequalities.

2.3 Nonsmooth systems versus hybrid systems

The nonsmooth dynamical systems we are dealing with, have a nonempty intersection with hybrid systems and cyber-physical systems, as is briefly discussed in Sect. 3.2.1. Like in hybrid systems, nonsmooth dynamical systems define continuous-time dynamics that can be identified with modes separated by guards, defined by the constraints. However, the strong mathematical structure of nonsmooth dynamical systems allows us to state results on the following points:

(i) Mathematical concept of solutions: well-posedness (existence, and possibly, uniqueness properties, (dis)continuous dependence on initial conditions).

(ii) Dynamical systems theoretic properties: existence of invariants (equilibria, limit cycles, periodic solutions,...) and their stability, existence of oscillations, periodic and quasi-periodic solutions and propagation of waves.

(iii) Control theoretic properties: passivity, controllability, observability, stabilization, robustness.

These latter properties, that are common for smooth nonlinear dynamical systems, distinguish the nonsmooth dynamical systems from the very general definition of hybrid or cyber-physical systems (Alur et al., 1995; Henzinger, 1996). Indeed, it is difficult to give a precise mathematical concept of solutions for hybrid systems since the general definition of hybrid automata is usually too loose.

2.4 Numerical methods for nonsmooth dynamical systems

To conclude this brief exposition of nonsmooth dynamical systems, let us recall an important fact related to numerical methods. Beyond their intrinsic mathematical interest, and the fact that they model real physical systems, using nonsmooth dynamical systems as a model is interesting, because there exists a large set of robust and efficient numerical techniques to simulate them. Without entering into the finer details, let us give two examples of these techniques:

- *Numerical time integration methods: convergence, efficiency (order of consistency, stability, symplectic properties).* For the nonsmooth dynamical systems described above, there exist event-capturing time-stepping schemes with strong mathematical results. These schemes have the ability to numerically integrate the initial value problem without performing an event location, but by capturing the event within a time step. We call an event, or a transition, every change into the index set of the active constraints in the complementarity formulation or in the normal cone inclusion. Hence these schemes are able to simulate systems with a huge number of transitions or even with finite accumulation of events (Zeno behavior). Furthermore, the schemes do not suffer from the weaknesses of the standard schemes based on a regularization (smoothing) of the multi-valued mapping resulting in stiff ordinary differential equations. For the time-integration of the initial value problem (IVP), or Cauchy problem, a lot of improvements of the standard time-stepping schemes for nonsmooth dynamics (Moreau–Jean time-stepping scheme) have been proposed in the last decade, in terms of accuracy and dissipation properties (Acary, 2012, 2013, 2016; Brüls, Acary, and Cardona, 2014; Capobianco and Eugster, 2018; Chen, Acary, Virlez, and Brüls, 2013; Schindler and Acary, 2013; Schindler, Rezaei, Kursawe, and Acary, 2015; Studer, 2009). A significant number of these schemes have been developed by members of the BIPOP team and have been implemented in the Siconos software (see Sect. 5.1).
- *Numerical solution procedure for the time–discretized problem, mainly through well-identified problems studied in the optimization and mathematical programming community.* Another very interesting feature is the fact that the discretized problem that we have to solve at each time–step is generally a well-known problem in optimization. For instance, for LCSs, we have to solve a linear

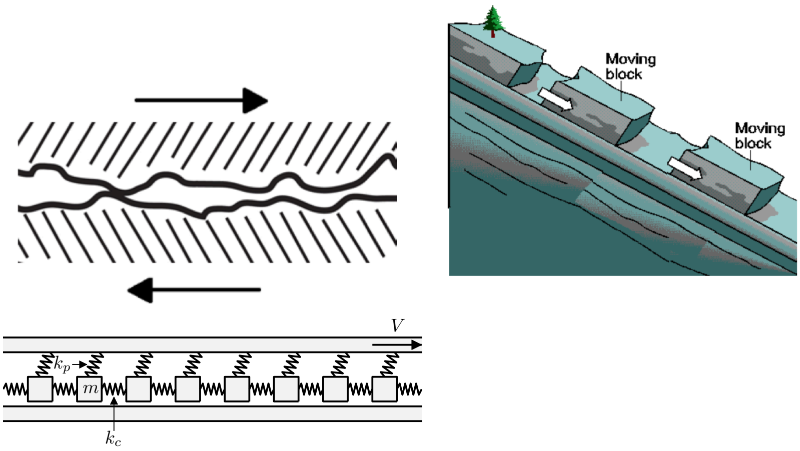
complementarity problem (Cottle, Pang, and Stone, 1992) for which there exist efficient solvers in the literature. Compared to the brute force algorithm with exponential complexity that consists of enumerating all the possible modes, the algorithms for linear complementarity problem have polynomial complexity when the problem is monotone.

2.5 Application fields

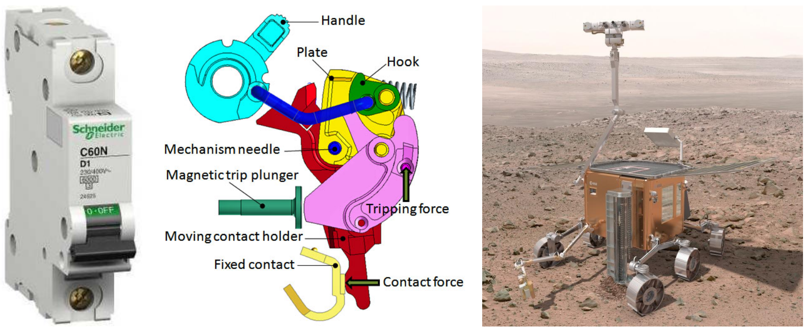
Nonsmooth dynamical systems arise in many application fields. We briefly highlight here some applications that have been treated in the BIPOP team and that we will continue in the TRIPOP team, as a validation for the research axes and also in terms of transfer (Sect. 5.3).

In mechanics, the main instances of nonsmooth dynamical systems are multibody systems with Signorini's unilateral contact, set-valued (Coulomb-like) friction and impacts, or in continuum mechanics, ideal plasticity, fracture or damage. Some illustrations are given in Figure 3(a-f). Other instances of nonsmooth dynamical systems can also be found in electrical circuits with ideal components (see Figure 3(g)) and in control theory, mainly with sliding mode control and variable structure systems (see Figure 3(h)). More generally, every time a piecewise, possibly set–valued, model of systems is invoked, we end up with a nonsmooth system. This is the case, for instance, for hybrid systems in nonlinear control or for piecewise linear modelling of gene regulatory networks in mathematical biology (see Figure 3(i)). Another common example of nonsmooth dynamics is also found when the vector field of a dynamical system is defined as a solution of an optimization problem under constraints, or a variational inequality. Examples of this kind are found in optimal control theory, in dynamic Nash equilibrium or in the theory of dynamic flows over networks.

(a) Rockfall (Bourrier, Dorren, Nicot, Berger, and Darve, 2009; Bourrier, Berger, Tardif, Dorren, and Hungr, 2012; Dupire et al., 2016), granular and debris flows

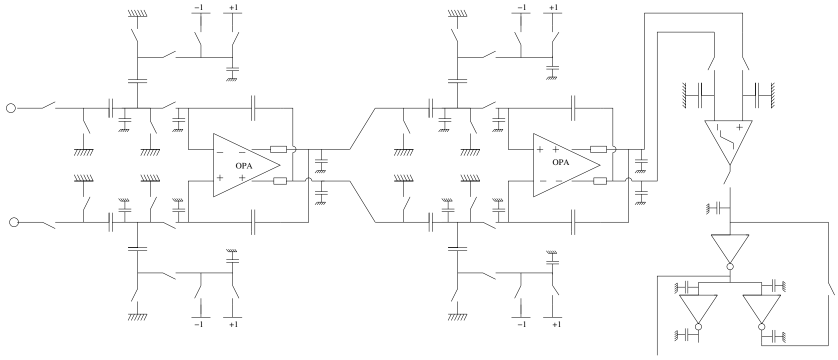


(b) Frictional interface and solitary waves in the Burridge-Knopoff model (Morales, James, and Tonnelier, 2016)

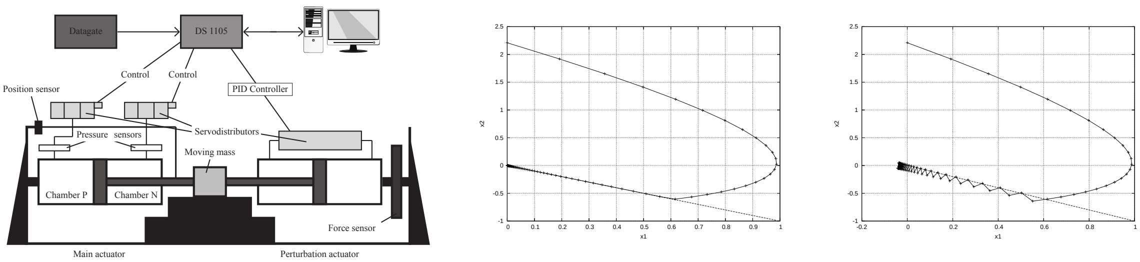


(c) Circuit breakers mechanisms (Akhadkar, Acary, and Brogliato, 2017) and Robots (ESA ExoMars Rover (Acary, Brémond, Kapellos, Michalczyk, and Pissard-Gibollet, 2013))

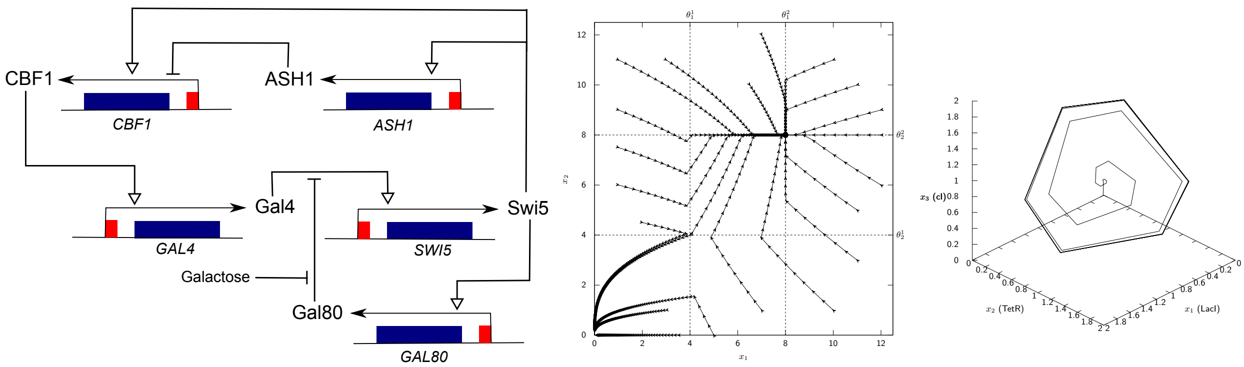
Figure 3: Application fields of nonsmooth dynamics (mechanics)



(d) Switched electrical circuits (delta-sigma converter) (Acary, Bonnefon, and Brogliato, 2011)



(e) Sliding mode control (Acary, Brogliato, and Orlov, 2012; Acary and Brogliato, 2010; Huber, Brogliato, Acary, Boubakir, Plestan, and B., 2016; Huber, Acary, Brogliato, and Plestan, 2016; Miranda-Villatoro, Brogliato, and Castanos, 2017)



(f) Gene regulatory networks (Acary, Jong, and Brogliato, 2013)

Figure 3: Application fields of nonsmooth dynamics (continued)

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3 Scientific objectives

In this section, we develop our scientific program. In the framework of nonsmooth dynamical systems, the activities of the project–team will be focused on the following research axes:

- **Axis 1**: Nonsmooth simulation and numerical modelling for natural gravitational risk in mountains. (detailed in Sect. 3.1).
- **Axis 2**: Modelling, simulation and control (detailed in Sect. 3.2).

These research axes will be developed with a strong emphasis on the software development and the industrial transfer that are detailed respectively in Sect. 5.1 and Sect. 5.3.

3.1 Axis 1: Nonsmooth simulation and numerical modelling for natural gravitational risk in mountains.

Participants: V. Acary, F. Bourrier, B. Brogliato, N. Collins-Craft, O. Goury.

Figure 4: The oldest known image of the fall of Mount Granier. Wood engraving (104 x 83 mm) extracted from the *Liber chronicarum* de Hartmann Schebel, Nuremberg, Anton Koberger 1493

In this research axis, we propose, on the one hand, to extend existing methods of simulation in mechanics of complex flows in a nonsmooth framework, which allows us to simplify the models by decreasing the physical parameters, and to make the numerical simulations more robust and thus to make possible the construction of reduced models or meta-models. On the other hand, the so-called "data-driven modelling" methods will be explored for gravity flows and prevention structures. The aim is to make the most of laboratory and observational data in order to build and calibrate the models, to evaluate their sensitivity, to improve their predictive character, *i.e.* to control and take into account the uncertainties, thanks to variational, statistical and AI methods.

This work will be conducted in close collaboration with the UR ETNA of INRAE as well as other researchers from INRIA (AIRSEA, LEMON). More generally, our collaboration with INRAE opens new long term perspectives on granular flow applications such as debris and mud flows, granular avalanches and the design of structural protections. The numerical methods that go with these new modelling approaches will be implemented in our software Siconos (see Sect. 5.1).

This research is also part of the more general context of a digital platform on environmental risk in the mountains, including intensive and cloud computing.

3.1.1 Rockfall trajectory modelling

Trajectory analysis of falling rocks during rockfall events is limited by the currently unrefined modelling of the impact phase (Bourrier, Dorren, Nicot, Berger, and Darve, 2009; Bourrier, Berger, Tardif, Dorren, and Hungr, 2012; Leine, Schweizer, Christen, Glover, Bartelt, and Gerber, 2014). The goal of this axis is to improve reliability of simulation techniques.

- *Rock fracturing:* When a rock falls from a steep cliff, it stores a large amount of kinetic energy that is partly dissipated though the impact with the ground. If the ground is composed of rocks and the kinetic energy is sufficiently high, the probability of the fracture of the rock is high and yields an extra amount of dissipated energy but also an increase of the number of blocks that fall. In this topic, we want to use the capability of the nonsmooth dynamical framework for modelling cohesion and fracture (Acary and Monerie, 2006; Jean, Acary, and Monerie, 2001) to propose new cohesive zone models with contact and friction.
- *Rock/forest interaction:* To prevent damage and incidents to infrastructure, a smart use of the forest is one of the ways to control trajectories (decrease of the run-out distance, jump heights and the energy) of the rocks that fall under gravity (Dorren et al., 2007; Dupire et al., 2016). From the modelling point of view and to be able to improve the protective function of the forest, an accurate modelling of impacts between rocks and trees is required. Due to the aspect ratio of the trees, they must be considered as flexible bodies that may be damaged by the impact. This new aspect offers interesting modelling research perspectives, expecially, building rockfall simulation method with mechanical models of trees including damage, fracture and plasticity.
- *Experimental validation:* The participation of INRAE with F. Bourrier makes possible the experimental validation of models and simulations through comparisons with real data. INRAE has extensive experience of lab and in-situ experiments for rockfall trajectory modelling (Bourrier, Dorren, Nicot, Berger, and Darve, 2009; Bourrier, Berger, Tardif, Dorren, and Hungr, 2012). It is a unique opportunity to strengthen our model and to prove that nonsmooth modelling of impacts is reliable for such experiments and forecasting of natural hazards.

3.1.2 Modelling and simulation of gravity hazards (debris flows, avalanches and large-scale rock flows)

Different modelling approaches are used in the literature depending on the type of hazard. For rockfalls and dense snow avalanches, methods that explicitly model the particles of granular materials (notably Discrete Element Methods - DEM) are preferred, whereas for flows (debris flows, avalanches and large-scale rockfalls), methods that assimilate the large number of individual constituents to materials with complex rheology are more commonly used (notably Material Point Method - MPM, Smoothed-Particle Hydrodynamics - SPH, Shallow Water models - SWM). It should be noted that these methods are most often explicit and regularize the constraints of inequalities and thresholds.

This research item will develop the following points:

- *Rethinking DEM, MPM, SPH and SWM methods in the nonsmooth framework.* This will allow a simple and efficient modelling of threshold and inequality phenomena (one-sided contact, impact with Coulomb friction, threshold behavior laws such as plasticity, damage or fracture, Bingham-type fluids) in order to develop new, implicit and robust numerical methods, where the most important physical features of frictional cohesive materials are well-modelled neglecting the second order phenomena. In a context of data utilization and prediction, these methods seem particularly well suited as our first experiments on block trajectography and rock flows have already shown.

- *Couple these methods to integrate the "multi-scale (micro/meso/macro)" character* of these problems or, more simply, to spatially couple at the same scale several physical phenomena better taken into account by different methods, for example a debris flow containing a material with complex rheology (MPM or SPH) and large size particles (DEM)

- *Use "data-driven mechanics" (Kirchdoerfer and Ortiz, 2016) and machine learning approaches* that are adapted to allow the enforcement of nonsmooth mechanics principles when behavioral models are not reliable and faithful to the observed physical phenomena. These techniques can also be used to model "sub-mesh" phenomena, which are not or only slightly taken into account in large-scale phenomenological models. This approach can be used to pass more refined nonsmooth models to the larger scale for applications.

- *Extend existing nonsmooth models to take into account more physics.* The existing nonsmooth mechanics models developed within the team and elsewhere use quite simple laws where possible, and currently only take into account fracture and Coulomb friction. However, particularly in the context of climate change, there is also interest in including temperature effects in models, which can substantially affect both fracture and frictional properties. There is also interest in developing nonsmooth versions of "rate and state" friction laws, which are very popular in the earthquake modelling community, but can also be applied to frictional contact more generally.

3.1.3 Data-driven modeling for prediction and mitigation of gravity hazards

The objective is to develop simplified models that can be used extensively for the development of calibration and uncertainty quantification methods that allow for the joint use of data from various sources to evaluate and improve the predictive capacity of gravity hazard models.
Using data-driven modeling methods (Kutz, 2013), the following points will be developed:

- *Statistical models* integrating various types of data and the hazard models developed in the previous section. The identification of the parameters of these hazard models, in particular using Bayesian approaches, will also allow the calibration and quantification of the uncertainties associated with the hazard models.

- *Model reduction approaches* (POD, PGD,...) or construction of substitution models (Sparse Polynomial Chaos, Gaussian Processes,...) to build simplified models usable in this context.

- *Application of different data assimilation techniques* (particle filters or variational methods) on the models described in the first axis and the reduced order models.

3.2 Axis 2: Modelling, simulation and control of non-smooth dynamical systems.

Participants: V. Acary, F. Bourrier, B. Brogliato, O. Goury, G. James, F. Miranda Villatoro, A. Tonnelier

This axis is dedicated to the modelling and the mathematical analysis of nonsmooth dynamical systems. It consists of two main directions: 1) Modelling, analysis and numerical methods and 2) Automatic control.

3.2.1 Modelling, analysis and numerical methods

Multibody vibro-impact systems

- *Multiple impacts with or without friction (short-term):* there are many different approaches to model collisions, especially simultaneous impacts (so-called multiple impacts)(Nguyen and Brogliato, 2014). One of our objectives is on one hand to determine the range of application of the models (for instance, when can one use "simplified" rigid contact models relying on kinematic, kinetic or energetic coefficients of restitution?) on typical benchmark examples (chains of aligned beads, rocking block systems). On the other hand, we will try to take advantage of the new results on nonlinear wave phenomena, to better understand multiple impacts in 2D and 3D granular systems. The study of multiple impacts with (unilateral) nonlinear visco-elastic models (Simon–Hunt–Crossley, Kuwabara–Kono), or visco-elasto-plastic models (assemblies of springs, dashpots and dry friction elements), is also a topic of interest, since these models are widely used.
- *Artificial or manufactured or ordered granular crystals, meta-materials (short-term):* Granular metamaterials (or more general nonlinear mechanical metamaterials) offer many perspectives for the passive control of waves originating from impacts or vibrations. The analysis of waves in such systems is delicate due to spatial discreteness, nonlinearity and non-smoothness of contact laws (James, 2012; James, Kevrekidis, and Cuevas, Physica D; Liu, James, Kevrekidis, and

Vainchtein, 2016; Porter, Kevrekidis, and Daraio, 2015). We will use a variety of approaches, both theoretical (e.g. bifurcation theory, modulation equations) and numerical, in order to describe nonlinear waves in such systems, with special emphasis on energy localization phenomena (excitation of solitary waves, fronts, breathers).

- *Systems with clearances, modelling of friction (long-term):* joint clearances in kinematic chains deserve specific analysis, especially concerning friction modelling (Akhadkar, Acary, and Brogliato, 2017). Indeed contacts in joints are often conformal, which involve large contact surfaces between bodies. Lubrication models should also be investigated.
- *Painlevé paradoxes (long-term):* the goal is to extend the results in Génot and Brogliato (1999), which deal with single-contact systems, to multi-contact systems. One central difficulty here is the understanding and the analysis of singularities that may occur in sliding regimes of motion.

As a continuation, our software Siconos (see Sect. 5.1) will be our favored software platform for the integration of these new modelling results.

Systemic risk

- The high consumption of natural resources by our society puts in question its long-term sustainability. The decrease of natural resources results in a deterioration of human welfare with a risk of society instability. Recently, a simple nature-society interrelations model, called the HANDY model (Human And Nature DYnamics), has been proposed by Montesharrei *et al* (2014) to address this concern with a special emphasis on the role of the stratification of the society. The Handy model is a four dimensional nonlinear dynamical system that describes the evolution of population, resources and accumulated wealth. We analyse the dynamics of this model and we explore the influence of two parameters: the nature depletion rate and the inequality factor. We characterize the asymptotic states of the system through a bifurcation analysis and we derive several quantitative results on the trajectories. We show that some collapses are irreversible and, depending on the wealth production factor, a bistability regime between a sustainable equilibrium and cycles of *collapse-and-regeneration* can be obtained. We discuss possible policies to avoid dramatic scenarios.

Cyber-physical systems (hybrid systems) Participants: V. Acary, B. Brogliato, C. Prieur, A. Tonnelier

Nonsmooth systems have a non-empty intersection with hybrid systems and cyber–physical systems. However, nonsmooth systems enjoy strong mathematical properties (concept of solutions, existence and uniqueness) and efficient numerical tools. This is often the result of the fact that nonsmooth dynamical systems are models of physical systems, and so can take advantage of their intrinsic properties (conservation or dissipation of energy, passivity, stability). A standard example is a circuit with n ideal diodes. From the hybrid point of view, this circuit is a piecewise smooth dynamical system with 2^n modes, that can be quite cumbersome to enumerate in order to determinate the current mode. As a nonsmooth system, this circuit can be formulated as a complementarity system for which there exist efficient time-stepping schemes and polynomial time algorithms for the computation of the current mode. The key idea of this research action is to benefit from this observation to improve hybrid system modelling tools.

• *Structural analysis of multimode DAE :* When a hybrid system is described by a Differential Algebraic Equation (DAE) with different differential indices in each continuous mode, the structural analysis has to be completely rethought. In particular, the re-initialization rule, when a switching occurs from one mode to another, has to be consistently designed. We propose in this action to use our knowledge in complementarity and (distribution) differential inclusions (Acary, Brogliato, and Goeleven, 2008) to design consistent re-initialization rules for systems with nonuniform relative degree vector $(r_1, r_2, \dots, r_m)$ and $r_i \neq r_j, i \neq j$.

• *Cyber–physical in hybrid systems modelling languages :* Nowadays, some hybrid modelling languages and tools are widely used to describe and to simulate hybrid systems (MODELICA, SIMULINK, and see Carloni, Passerone, Pinto, and Angiovanni-Vincentelli (2006) for references therein). Nevertheless, the compilers and the simulation engines behind these languages and tools suffer from several serious weaknesses (failure, nonsensical output or extreme sensitivity to simulation parameters), especially when some components, that are standard in nonsmooth dynamics, are introduced (piecewise smooth characteristic, unilateral constraints and complementarity condition, relay characteristic, saturation, dead zone, ...). One of the main reasons is the fact that most of the compilers reduce the hybrid system to a set of smooth modes modelled by differential algebraic equations and some guards and reinitialization rules between these modes. Sliding mode and Zeno-type behaviour are extremely difficult for hybrid systems and relatively simple for nonsmooth systems. With B. Caillaud (Inria HYCOMES) and M. Pouzet (Inria PARKAS), we propose to improve this situation by implementing a module able to identify/describe nonsmooth elements and to efficiently handle them with SICONOS as the simulation engine. They have already carried out a first implementation (Caillaud, 2014) in Zelus, a synchronous language for hybrid systems http://zelus.di.ens.fr. Removing the weaknesses related to the nonsmoothness of solutions should improve hybrid systems towards robustness and certification.

- *A general solver for piecewise smooth systems* This direction is the continuation of the promising result on modelling and the simulation of piecewise smooth systems (Acary, Jong, and Brogliato, 2013). As for general hybrid automata, the notion or concept of solutions is not rigorously defined from the mathematical point of view. For piecewise smooth systems, multiplicity of solutions can happen and sliding solutions are common. The objective is to recast general piecewise smooth systems in the framework of differential inclusions with Aizerman–Pyatnitskii extension (Acary, Jong, and Brogliato, 2013; Filippov, 1988). This operation provides a precise meaning to the concept of solutions. Starting from this point, the goal is to design and study an efficient numerical solver (time-integration scheme and optimization solver) based on an equivalent formulation as mixed complementarity systems of differential variational inequalities. We are currently discussing the issues in the mathematical analysis. The goal is to prove the convergence of the time-stepping scheme to get an existence theorem. With this work, we should also be able to discuss the general Lyapunov stability of stationary points of piecewise smooth systems.

Numerical optimization for discrete nonsmooth problems

- Second Order Cone Complementarity Problems (SOCCP) for discrete frictional systems (short-term): After some extensive comparisons of existing solvers on a large collection of examples (Acary, Brémond, Koziara, and Pérignon, 2014; Acary, Brémond, and Huber, 2018), the numerical treatment of constraint redundancy by the proximal point technique and the augmented Lagrangian formulation seems to be a promising path for designing new methods. From the comparison results, it appears that the redundancy of constraints prevents the use of second order methods such as semi-smooth Newton methods or interior point methods. With P. Armand (XLIM, U. de Limoges), we propose to adapt recent advances for regularizing constraints for the quadratic problem (Friedlander and Orban, 2012) for the second-order cone complementarity problem.
- The other question is the improvement of the efficiency of the algorithms by using accelerated schemes for the proximal gradient method that come from large-scale machine learning and image processing problems. Learning from the experience in large-scale machine learning and image processing problems, the accelerated version of the classical gradient algorithm (Nesterov, 1983) and the proximal point algorithm (Beck and Teboulle, 2009), and many of their further extensions, could be of interest for solving discrete frictional contact problems. Following the visit of Y. Kanno (University of Tokyo) and his preliminary experience on frictionless problems, we will extend its use to frictional contact problem. When we face large-scale problems, the main available solvers is based on a Gauss–Seidel strategy that is intrinsically sequential. Accelerated first-order methods could be a good alternative to benefit from distributed scientific computing architectures.

3.2.2 Automatic Control

This last item is dedicated to the automatic control of nonsmooth dynamical systems, or the nonsmooth control of smooth systems. The first research direction concerns the discrete-time sliding mode control and differentiation. The second research direction concerns multibody systems with unilateral constraint, impacts and set-valued friction. The third research direction concerns a class of dynamics which is an extension of linear complementarity systems (or, equivalently, of differential algebraic equations).

Discrete-time Sliding-Mode Control (SMC), State Observers (SMSO) and Differentiators (SMD)

- *SMC with output feedback:* Output feedback can take different forms, like the use of observers/differentiators in the loop (specific dynamic output feedback), or the design of static or dynamic output feedback (without state observation). The time-discretization of such feedback systems and its analysis remains largely open.
- *Unifying algorithm for discrete-time SMC and SMD:* maximal monotone operators, proximal algorithms.

Control of nonsmooth discrete Lagrangian systems

- *Linear Complementarity Systems (LCS):* the PhD thesis of Aya Younes is dedicated to the trajectory tracking control in LCS. In particular the cases with uncertainties and with state jumps are carefully analysed. The PhD thesis of Quang-Hung Pham focuses on networks of LCS. In both cases passivity is a central tool for the analysis.
- *Optimal control:* the optimal control of mechanical systems with unilateral constraints and impacts, largely remains an open issue. Through a collaboration with Moritz Diehl (Freiburg University) the problem has been tackled using a suitable dynamics transformation of Lagrangian complementarity systems into a Filippov "classical" differential inclusion with absolutely continuous solutions. The results are restricted to a single unilateral frictionless constraint. The global objective is to enlarge it to multiple unilateral constraints (hence multiple impacts) and friction.
- *Cable-driven systems:* these systems are typically different from the cable-car systems, and are closer in their mechanical structure to so-called tensegrity structures. The objective is to actuate a system *via* cables supposed in the first instance to be flexible (slack mode) but non-extensible in their longitudinal direction. This gives rise to complementarity conditions, one big difference with usual complementarity Lagrangian systems being that the control actions operate directly in one of the complementary variables (and not in the smooth dynamics as in cable-car systems). Therefore both the cable models and the control properties are expected to differ a lot from what we may use for cableway systems (for which guaranteeing a positive cable tension is usually not an issue, hence avoiding slack modes, but the deformation of the cables due to the nacelles and cables weights, is an important factor). Tethered systems are a close topic.
- *Robot-object underactuated dynamical systems:* such systems are made of a controlled part (called the robot) and an uncontrolled part (called the object). Both are linked by Lagrange multipliers which represent the contact forces. The object can be controlled only through the multipliers, which are in turn a function of the system's state. Examples are bipeds which walk, run, jump, juggling, tapping, pushing robots, prehensile and non-prehensile tasks, some cable-driven systems, and some circuits with nonsmooth set-valued components. A global approach consists in a backstepping-like control strategy. The goal is to derive a unifying framework which can be easily adapted to all these various systems and tasks.

Switching LCS and DAEs

- We have gained a strong experience in the field of complementarity systems and distribution differential inclusions (Acary, Brogliato, and Goeleven, 2008; Brogliato and Thibault, 2010), that may be seen as some kind of switching DAEs. More recently we have obtained preliminary results on the analysis of so-called differential-algebraic linear complementarity systems (DALCS) and descriptor-variable LCS (DVLCS), as well as on switching DAEs with state-dependent swithing bilateral constraints. These systems can be seen as DAEs with added complementarity constraints, or as LCS with added equality constraints, or as DAEs with nonsmooth equality constraints. Their well-posedness (existence and uniqueness of solutions to the one-step nonsmooth problem of the implicit Euler scheme, existence and uniqueness of solutions to the continuous-time system) is non-trivial. The case of systems with state-jumps also requires careful analysis.
- A closely related subject is that of interconnections of LCSs or extensions of these (like differential inclusions with maximal monotone properties). The stability of the interconnected system is a topic of interest, as well as, the resulting collective behavior.

Dynamics of complex nonlinear networks, set-valued couplings

- The interconnections of uncertain dynamical systems is a topic of broad interest within the control community. For the case of nonlinear agents with set-valued coupling laws, many questions remain open regarding the resulting behavior of the network, as well as, their robustness properties against parametric uncertainties and external disturbances. The PhD thesis of Quang-Hung Pham focuses on such issues within the context of robust synchronization of LCSs.

- Recently, novel extensions of the concept of passivity have been studied for the analysis of systems away from equilibrium (Miranda-Villatoro, Forni, and Sepulchre, 2022). However, their relevance in the context of networks remains largely unexplored.
- Two-dimensional networks of oscillators with set-valued generalized Coulomb friction laws arise challenging questions regarding their dynamics (nonlinear oscillations, localized waves, excitability), with applications in earthquake dynamics and friction control.
- G. James has recently introduced in collaboration with F. Karbou (Centre d'Etudes de la Neige, Grenoble) a nonsmooth dynamical system on a network suitable for segmenting wet snow areas in SAR (synthetic-aperture radar) satellite images. The network corresponds to a large ensemble of pixels of a grayscale image, whose evolutions are coupled or uncoupled depending on their distance and local topography given by a digital elevation model. This yields an excitable dynamical system that tends to create domain walls surrounding snowy areas. The system provides very good identification results and arises nontrivial questions regarding its theoretical analysis, optimization (parameters, complexity) and generalizations.

4 Social and environmental responsibility

As for the environmental footprint, we have already decided to drastically reduce our air travel and our participation in international conferences. For instance, trips of less than 10 hours by train should not be made by plane. International conferences should be coupled with a visit to colleagues or other scientific events. Concerning the computer equipment, it is not replaced before 5 years and we try to keep the office machines between 7 and 10 years.

Regarding the social impact, the emergence of the research axis 1 on natural gravitational hazards in relation to climate change and studies on systemic risk are a way to focus research on the major concerns of societies. Industrial collaborations are now also evaluated according to the social and environmental responsibility efforts of the partners.

The question of the social and environmental footprint and impact of our research is discussed in more detail at our annual team seminar.

5 Software development and industrial transfer

5.1 Software: SICONOS

The aim of this development is to provide a common platform for the modelling, simulation, analysis and control of general nonsmooth dynamical systems. Besides usual quality attributes for scientific computing software, we want to provide a common framework for various scientific fields, to be able to rely on the existing developments (numerical algorithms, description and modelling software), to support exchanges and comparisons of methods, to disseminate the know-how to other fields of research and industry, and to take into account the diversity of users (end-users, algorithm developers, framework builders) in building user interfaces in Python and industry oriented applications.

In the framework of the FP5 European project Siconos (2002-2006), V. Acary was the leader of the Work Package 2 (WP2), dedicated to the numerical methods and the software design for nonsmooth dynamical systems. This gave rise to the platform Siconos in 2004 which was the main software development task in the Bipop team. We invested an important part of our activities developing new algorithms and maintaining the software architecture to answer to new challenges that come from applications. Siconos is now a mature software that can be used as a stand-alone software, as a module in Python or as a computational engine or library inside a package dedicated to a specific community. The software consists of around 370,000 lines of codes in C++, C, Fortran 77 and Python distributed under the Apache 2.0 license on Github. The goal is to continue this development in TRIPOP.

Users community and assessment Siconos is used for research, education and by industrial partners with approximately 30 frequent users (200 registered users on the user-list "siconos-users"). In the Tripop Team, a large number of publications and also PhD theses and post-doc fellows use SICONOS. For instance, in control papers, there is no alternative to SICONOS for the simulation of our systems. Let us list the main use cases in other teams:

- Education : SICONOS is used for teaching simulation of nonsmooth systems at the University of Limoges.
- Research: B. Caillaud (Hycomes team) for simulation of hybrid and cyber-physical systems, S. Adly and H. Massias (XLIM, University of Limoges) for simulation of electrical circuits. SICONOS has been coupled to MBDyn (main open-source software for the simulation of multi-body-systems), P. Masareti and M. Fancello (Politecnico de Milano). C. Touzé (IMSIA, ENSTA ParisTech), C. Issanchou (Institut Jean Le Rond D'Alembert/UPMC) have used SICONOS for the simulation of strings with impacts in musical instruments to synthesize digital sounds. M. Ferris and O. Huber (University of Madison, Wisconsin) have used SICONOS to develop new algorithms for contact and friction based on the PathVI approach. R. Kikuuwe (Kyushu University, Japan) have used SICONOS for control and simulation of nonsmooth robotic systems, Gazebo users. Through the work of S. Sinclair in the ADT Rope[1](Inria technological development action), SICONOS is coupled to GAZEBO[2] which is now a standard de facto for the simulation of robotic systems inside ROS (Robot Operating system[3]). Siconos is used at INRAE for the simulation of rock impacts on masonry and thought the software PlatRock[4] for the simulation of rockfall trajectory. In a collaboration with Ngoc Son NGUYEN (Univ Nantes, GEM) , Siconos is coupled with a SPH code for the modelling and simulation of riprap protection on the shoreline.
- Industry: Schneider Electric uses SICONOS in production for virtual prototyping of circuit breakers, and in particular, for the robustness analysis of circuit breakers to manufacturing tolerances, Trasys Space for the simulation of the ExoMars Rover[5] of European Space Agency (Acary, Brémond, Kapellos, Michalczyk, and Pissard-Gibollet, 2013), Electricité de France uses SICONOS in the Saladyn project to simulate hydraulic dams made of concrete and rocky blocks, $\varphi-$ Ingeneria (Chile) uses SICONOS for the simulation of flows of granular material in ore processes. Géolithe uses Siconos for the simulation of prevention structure to rockfall avalanches.

SICONOS also substantially increases our visibility. Without SICONOS, it would have been difficult to get our collaboration with ANSYS and to continue with Schneider Electric, and all-day use of the R&D engineers. In the same spirit, the book on switched electrical circuits and partly the book on numerical methods would not have been written. Up to now, this software is mainly used by experts through various interfaces. Our goal is to extend its use by interfacing SICONOS with several open-source software codes that are standard or widespread in their own community (GAZEBO, MODELICA compiler, FreeCAD).

Description of the state of the art and placement SICONOS is the only software that simulates general nonsmooth dynamical systems in a quite abstract form. This allows for applications from mechanical, electrical and control engineering to gene networks in biology. In computational mechanics, there are mainly three related research software codes.

- LMGC90 is a companion software of SICONOS for applications in mechanical engineering and multi-physics simulation. Now, we are seriously thinking on how to merge our development, to increase the critical mass and to create a consortium to respond to industrial requests.
- MBsim[6]. This software, mainly developed at TUM (Technische Universität München) provides a subset of functionalities of SICONOS concerning the simulation of multibody systems with contact and friction.
- CHRONO Engine[7] is an open-source tool for simulation of mechanical systems. Although the choice of solvers is quite restrictive, a massive investment by DARPA and NVidia through the SBEL

[1] http://www.inrialpes.fr/bipop/people/acary/publications/Misc/ROPE_adt_2015-final.pdf
[2] http://gazebosim.org
[3] http://www.ros.org
[4] http://platrock.org
[5] http://exploration.esa.int/mars/45084-exomars-rover/
[6] https://github.com/mbsim-env/mbsim
[7] http://chronoengine.info/chronoengine/

lab[8] in developing GUI rendering tools and GPU parallel computation renders this project very attractive. Unfortunately, the European context renders the same kind of development difficult for SICONOS. We hope to partly bridge this gap thanks to a consortium with LMGC90.

Priorities for the next 4 years The objectives for the development of functionalities in SICONOS are:

1. Functionalities
   - Space discretization and spectral methods of flexible mechanical systems in particular cable systems.
   - Numerical methods for rockfall trajectory (INRAe collaboration), geomaterials — mainly granular materials (Material Point Method and Finite Element Method) — for debris and mud flows, rock avalanches and the design of structural protections.
2. Architecture
   - Code modernization with a move to C++ 20 (namespace, metaprogramming, ldots)
   - redesign of the architecture to move from Array of structures(AoS) to Structure of Arrays (SoA) (memory access and optimization)
   - switch to HPC (massive parallelization)

Software web site:

- General web site: https://nonsmooth.gricad-pages.univ-grenoble-alpes.fr/siconos
- Developer's site: https://github.com/siconos/siconos
- Dashboard: http://siconos-dashboard.univ-grenoble-alpes.fr:8080/index.php?project=siconos
- Youtube channel: https://www.youtube.com/channel/UCgv2siTCJeSdWFPTDk71Iyw

5.2 Software: Platrock

Platrock is a software dedicated to the simulation of block propagation accessible via a web browser. It aims to transfer knowledge to practioners and to provide them with accurate simulation tools. To date, 50 accounts have been created to access the software platform and about 10 accounts are regularly used by private (SNCF, Géolithe, Alp'géorisques) or public practitioners (ONF/RTM in particular).

This software, initially developed in INRAE, is written in Python and uses Siconos as a library. It allows the simulation of block propagation in 2D and 3D using several representative approaches from the different existing modelling tools in the literature, among which the nonsmooth approach, developed in Tripop, is the most advanced one.

- Software Family
  1. `transfer`: Transfer software.
- Audience:
  1. `community`: large audience software, usable by people inside and outside the field with a clear and strong dissemination, validation, and support action plan;
- Evolution and maintenance:
  1. `lts`: long term support.
- Duration of the Development (Duration):
  3 years. First developments in 2018.

Software's web site: https://gitlab.com/platrock/platrock

5.3 Industrial transfer

Cableways transportation (STRMTG)
Participants: V. Acary, B. Brogliato, M. Brémond, F. Pérignon
We have recently contacted both STRMTG and the POMA company about modelling, simulation and

[8] http://sbel.wisc.edu

control of cable-transport systems. In such systems, the question of the coupling between the nonlinear dynamics of cables and their supports with unilateral contact and friction appears now to be the determinant in order to increase the performance of the cableway systems, especially for urban transportation systems.

Natural hazards (Géolithe, INDURA)
Participants: F. Bourrier, V. Acary,
Through a starting collaboration with F. Bourrier (INRAE Grenoble), we will use our software SICONOS to assess the natural risk related to rockfalls and the rock slope stability. We work with several socio-economic actors through the OCIRN and SMART PROTECT projects to set up an industrialization of our simulation and calculation techniques in an operational context. For this, the creation of a consortium around Platrock and Siconos is under study. The industrialization part (marketing, maintenance, support) will be ensured by the company HALIAS technologies.

6 Institutional context and positioning

6.1 Positioning inside Inria

We are the only team at Inria dealing all these aspects of nonsmooth systems with strong skills on complementarity problems and variational inequalities, their analysis, control and numerical simulation. As far as we know, the issue of modelling and simulation of natural hazards in mountains is not addressed by other teams.

Strong collaborations We list here the INRIA project teams with which we are actively working:

- HYCOMES (B.HYCOMES (B. Caillaud, K. Ghorbal, A. Benveniste), INRIA Rennes Bretagne Atlantique and PARKAS (M. Pouzet, T. Bourke), INRIA Paris & ENS Paris (Modeliscale).
  In the framework of the Inria Project Lab (IPL) and the FUI Modeliscale, we strongly collaborate with HYCOMES (Inria Rennes Atlantique, B. Cailllaud), PARKAS (Inria Paris, M. Pouzet) on the analysis, simulation of cyberphysical systems.
- VALSE (D.Efimov, A.Polyakov)
  We jointly work on the implicit discretization for sliding-mode control and observation. In 2018, an ANR project DIGITSLID (coordinator B. Brogliato) has been accepted to continue this effort.

Possible collaborations Other Inria teams with which our scientific activities have non void intersection are:

- HEPHAISTOS (Inria Sophia Antipolis): control of cable-driven systems.
- DEFROST (Inria Lille): control of deformable mechanical systems.
- POEMS (Inria Paris): wave propagation and nonlinear wave analysis.
- SPHINX (Inria Nancy): control and stabilization of partial differential equations.
- STEEP (Inria Grenoble): Systemic risk.
- AIRSEA (Inria Grenoble) : data assimilation, uncertainty analysis and data-driven modelling.
- LEMON (Inria Sophia Antipolis) : simulation and modelling of natural risks.
- CONCAUST (Inria Bordeaux/UC Davis exploratory action)

6.2 Local and national contexts

Again, we do not know other multi-disciplinary teams in France that work on all the aspects of nonsmooth dynamical systems (modelling, analysis, simulation and control) in a general setting.

In this section, we list the French laboratories that work on nonsmooth dynamical systems, however focusing on one aspect only. We classify them by scientific domains, and we specify laboratories with which we have collaborations, and the other that could be considered as future collaborators.

Mechanical engineering and computational mechanics We actively work with the following laboratories, which represent the main actors in the domain of mechanical systems with contact and friction:

- INRAe/IGE/ECRINS (G. Chambon, T.Faug, N. Eckert):Gravity hazards and the cryosphere in the mountains
- LTDS-ENTPE (C. Lamarque, A. Ture Savadkoohi): nonlinear dynamics of cables with contact and friction within the co-supervision of the PhD thesis of C. Bertrand.
- Geomas-Insa Lyon (D. Bertrand, S. Grange, C. Silvani):Application of the nonsmooth contact dynamics method to structures in Civil Engineering.
- GEM, École Centrale de Nantes (Ngoc Son NGUYEN): modelling and simulation of riprap protection on the shoreline

Control

- LAAS (A. Tanwani): analysis, control, state observation of set-valued dynamical systems, Lur'e set-valued systems.
- LS2N, École Centrale de Nantes (F. Plestan, Y. Aoustin, M. Ghanes): discrete time sliding–mode control.
- GEM, École Centrale de Nantes (I. Stefanou): Nonsmooth control of frictional sliding applied to earthquake mitigation

6.3 International context

The other groups which work on nonsmooth dynamical systems in the world are the University of Stuttgart, Mechanical Engineering (R.I. Leine), Technology University of Eindhoven (N. van de Wouw), University of Naples (A. Frasca, L. Iannelli), Universita di Parma (A. Tasora), Argonne National Laboratory, Mathematics and Computer Science Division (M. Anitescu) and University of Wisconsin at Madison, Department of Mechanical Engineering (D. Negrut). We know quite well all these groups, but are the unique one to deal with all aspects of nonsmooth dynamical systems from modelling to mathematical analysis. From the software point of view, our principal competitor is the Simulation-Based Engineering Lab (SBEL) of the University of Wisconsin at Madison with the software Chrono.

Main collaborations

- Canada: McGill University, Mechanical Engineering (M. Legrand, J. Kovecses): nonlinear modes in nonsmooth mechanical systems, numerical solvers for contact mechanics, impact mechanics. J. Kocveses was invited professor for 2 months in 2017 in the BIPOP team, and M. Legrand will be hosted 3 months in 2018 in the TRIPOP team.
- Israel: Technion, Mechanical Engineering (Y. Starosvetsky): nonlinear waves in discrete mechanical systems.
- Japan: Kyushu University (R. Kikuuwe) and Tokyo Institute of Technology (Y. Kanno): discrete-time sliding-mode control and observers, optimization of structures.
- Belgique: Université de Liège, Aerospace and Mechanical Engineering (O. Brüls): numerical analysis for nonsmooth systems and DAEs.
- Argentina: and Centro de Investigación de Métodos Computacionales (CIMEC), Sante FE, Argentina (A. Cardona): numerical analysis for nonsmooth systems and DAEs.
- Chile:
  - INRIA Chile
  - Universidad de Chile (H. Ramirez). Optimal control.
  - Universidad Adolfo Ibanez (R. Comineti, P Toledo, J.C. Silva, C. Caceres): Nonsmooth optimization and mines engineering.
  - Universidad Técnica Federico Santa María (Juan Peypouquet and Eduardo Cerpa))Optimal control of sweeping process and control of PDEs
- World-wide: participation to the creation of the first overlay journal in Mechanics, JTCAM. The Journal of Theoretical, Computational and Applied Mechanics is a scholarly journal, provided on

a Fair Open Access basis, without cost to both readers and authors. The Journal aims to select publications of the highest scientific calibre in the form of either original research papers or reviews.

These main collaborations are already granted by international projects or will be the object of applications to project calls.

One-off collaborations A non-exhaustive list of people whose scientific interests partly match with ours (those with whom we have or had collaborations are marked with (**)):

- (**) University of Stuttgart, Mechanical Engineering (R.I. Leine): nonsmooth mechanical systems analysis and control, impact mechanics.
- University of Groningen (K. Camlibel): Lur'e set-valued systems, dissipativity and complementarity.
- Technology University of Eindhoven (N. van de Wouw): control of nonsmooth systems.
- University of Bristol (A. Champneys, N. Hogan): analysis of Painlevé paradoxes.
- (**) University of Naples (A. Frasca, L. Iannelli): complementarity systems analysis and computation.
- Rensselaer Polytechnic Institute, Mathematics of robotics (J. Trinkle): complementarity in robotics.
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