Proposal for a new project team (EPC)

TRIPOP

Modeling, Simulation and Control of Nonsmooth Dynamical Systems

May 25, 2018

- **Domain** : Applied Mathematics, Computation and Simulation
- **Theme** : Optimization and control of dynamic systems
- **Associated institutions "établissements de rattachement"** : Inria and Grenoble-INP, CNRS through the Laboratoire Jean Kuntzman (LJK UMR CNRS 5524, Dir. Stéphane Labbé).

Abstract

This document is a proposal for a joint research team, TRIPOP, of Inria Grenoble Rhone-Alpes and of the Laboratoire Jean Kuntzmann. This new team is a follow up of the BIPOP team (2003–2017). The team is mainly concerned by the modeling, the mathematical analysis, the simulation and the control of nonsmooth dynamical systems. Nonsmooth dynamics concerns the study of the time evolution of systems that are not smooth in the mathematical sense, i.e., systems that are characterized by a lack of differentiability, either of the mappings in their formulations, or of their solutions with respect to time. In mechanics, the main instances of nonsmooth dynamical systems are multibody systems with Signorini unilateral contact, set-valued (Coulomb-like) friction and impacts. In Electronics, examples are found in switched electrical circuits with ideal components (diodes, switches, transistors). In Control, nonsmooth systems arise in the sliding mode control theory and in optimal control. A lot of examples can also be found in cyber-physical systems (hybrid systems), in transportation sciences, in mathematical biology or in finance.

1 General scope and motivations

Nonsmooth dynamics concerns the study of the time evolution of systems that are not smooth in the mathematical sense, *i.e.*, systems that are characterized by a lack of differentiability, either of the mappings in their formulations, or of their solutions with respect to time. The class of nonsmooth dynamical systems recovers a large variety of dynamical systems that arise in many applications. The term "nonsmooth", as the term "nonlinear", does not precisely define the scope of the systems we are interested in but, and most importantly, they are characterized by the mathematical and numerical properties that they share. To give more insight of what are nonsmooth dynamical systems, we give in the sequel a very brief introduction of their salient features. For more details, we refer to [37, 8, 22, 53, 33, 40, 14].

1.1 A flavor of nonsmooth dynamical systems

As a first illustration, let us consider a linear finite-dimensional system described by its state $x(t) \in \mathbb{R}^n$ over a time-interval $t \in [0, T]$:

$$\dot{x}(t) = Ax(t) + a, \quad A \in \mathbb{R}^{n\times n},\, a \in \mathbb{R}^n, \tag{1}$$

subjected to a set of m inequality (unilateral) constraints:

$$y(t) = Cx(t) + c \geq 0, \qquad C \in \mathbb{R}^{m\times n},\, c \in \mathbb{R}^m. \tag{2}$$

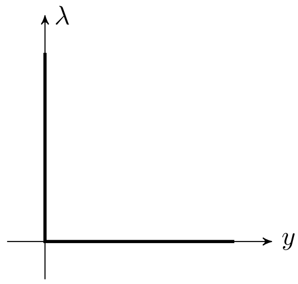


Figure 1: Complementarity condition $0 \leq y \perp \lambda \geq 0$.

If the constraints are physical constraints, a standard modeling approach is to augment the dynamics in (1) by an input vector $\lambda(t) \in \mathbb{R}^m$ that plays the role of a Lagrange multiplier vector. The multiplier restricts the trajectory of the system in order to respect the constraints. Furthermore, as in the continuous optimization theory, the multiplier must be signed and must vanish if the constraint is not active. This is usually formulated as a complementarity condition:

$$0 \leq y(t) \perp \lambda(t) \geq 0, \tag{3}$$

which models the one-sided effect of the inequality constraints. The notation $y \geq 0$ holds component–wise and $y \perp \lambda$ means $y^T\lambda = 0$. All together we end up with a Linear Complementarity System (LCS) of the form,

$$\begin{cases} \dot{x}(t) = Ax(t) + a + B\lambda(t) \\ y(t) = Cx(t) + c \\ 0 \leq y(t) \perp \lambda(t) \geq 0 \end{cases} \tag{4}$$

where $B \in \mathbb{R}^{n\times m}$ is the matrix that models the input generated by the constraints. In a more general way, the constraints may also involve the Lagrange multiplier,

$$y(t) = Cx(t) + c + D\lambda(t) \geq 0, \qquad D \in \mathbb{R}^{m\times m}, \tag{5}$$

leading to a general definition of LCS as

$$\begin{cases} \dot{x}(t) = A\,x(t) + a + B\,\lambda(t) \\ y(t) = C\,x(t) + c + D\,\lambda(t) \\ 0 \leq y(t) \perp \lambda(t) \geq 0. \end{cases} \tag{6}$$

The complementarity condition, illustrated in Figure 1 is the archetype of a nonsmooth graph that we extensively use in nonsmooth dynamics. The mapping $y \mapsto \lambda$ is a multi-valued (set-valued) mapping, that is nonsmooth at the origin. It has a lot of interesting mathematical properties and reformulations that come mainly from convex analysis and variational inequality theory. Let us introduce the indicator function of $\mathbb{R}_+$ as

$$\Psi_{\mathbb{R}_+}(x) = \begin{cases} 0 \text{ if } x \geq 0, \\ +\infty \text{ if } x < 0. \end{cases} \tag{7}$$

This function is convex, proper and can be sub-differentiated [43]. The definition of the subdifferential of a convex function $f : \mathbb{R}^m \to \mathbb{R}$ is defined as:

$$\partial f(x) = \{x^\star \in \mathbb{R}^m \mid f(z) \geq f(x) + (z-x)^\top x^\star, \forall z\}. \tag{8}$$

A basic result of convex analysis reads as

$$0 \leq y \perp \lambda \geq 0 \Longleftrightarrow -\lambda \in \partial\Psi_{\mathbb{R}_+}(y) \tag{9}$$

that gives a first functional meaning to the set-valued mapping $y \mapsto \lambda$. Another interpretation of $\partial\Psi_{\mathbb{R}_+}$ is based on the normal cone to a closed and nonempty convex set C:

$$N_C(x) = \{v \in \mathbb{R}^m | v^\top(z-x) \leq 0 \text{ for all } z \in K\}. \tag{10}$$

It is easy to check that $\partial\Psi_{I\!R_+} = N_{I\!R_+}(x)$ and it follows that

$$0 \leq y \perp \lambda \geq 0 \Longleftrightarrow -\lambda \in N_{I\!R_+}(y). \tag{11}$$

Finally, the definition of the normal cone yields a variational inequality:

$$0 \leq y \perp \lambda \geq 0 \Longleftrightarrow \lambda^\top(y-z) \leq 0, \forall z \geq 0. \tag{12}$$

The relations (11) and (12) allow one to formulate the complementarity system with $D = 0$ as a differential inclusion based on a normal cone (see (15)) or as a differential variational inequality. By extending the definition to other types of convex functions, possibly nonsmooth, and using more general variational inequalities, the same framework applies to the nonsmooth laws depicted in Figure 2 that includes the case of piecewise smooth systems.

The mathematical concept of solutions depends strongly on the nature of the matrix quadruplet (A, B, C, D) in (6). If D is a positive definite matrix (or a P-matrix), the Linear Complementarity problem

$$0 \leq Cx + c + D\lambda \perp \lambda \geq 0, \tag{13}$$

admits a unique solution $\lambda(x)$ which is a Lipschitz continuous mapping. It follows that the Ordinary Differential Equation (ODE)

$$\dot{x}(t) = Ax(t) + a + B\lambda(x(t)), \tag{14}$$

is a standard ODE with a Lipschitz right-hand side with a C^1 solution for the initial value problem. If $D = 0$, the system can be written as a differential inclusion in a normal cone as

$$-\dot{x}(t) + Ax(t) + a \in BN_{I\!R_+}(Cx(t)), \tag{15}$$

that admits a solution that is absolutely continuous if CB is a definite positive matrix and the initial condition satisfies the constraints. The time derivative $\dot{x}(t)$ and the multiplier $\lambda(t)$ may have jumps and are generally considered as functions of bounded variations. If $CB = 0$, the order of nonsmoothness increases and the Lagrange multiplier may contain Dirac atoms and must be considered as a measure. Higher–order index, or higher relative degree systems yield solutions in terms of distributions and derivatives of distributions [10].

A lot of variants can be derived from the basic form of linear complementarity systems, by changing the form of the dynamics including nonlinear terms or by changing the complementarity relation by other multivalued maps. In particular the nonnegative orthant may be replaced by any convex closed cone $K \subset I\!R^m$ leading to complementarity over cones

$$K^\star \ni y \perp \lambda \in K, \tag{16}$$

where $K^\star$ its dual cone given by

$$K^\star = \{x \in I\!R^m \mid x^\top y \geq 0 \text{ for all } y \in K\}. \tag{17}$$

In Figure 2, we illustrate some other basic maps that can used for defining the relation between λ and y. The saturation map, depicted in Figure 2(a) is a single valued continuous function which is an archetype of piece-wise smooth map. In Figure 2(b), the relay multi-function is illustrated. If the upper and the lower limits of λ are respectively equal to 1 and -1, we obtain the multivalued sign function defined as

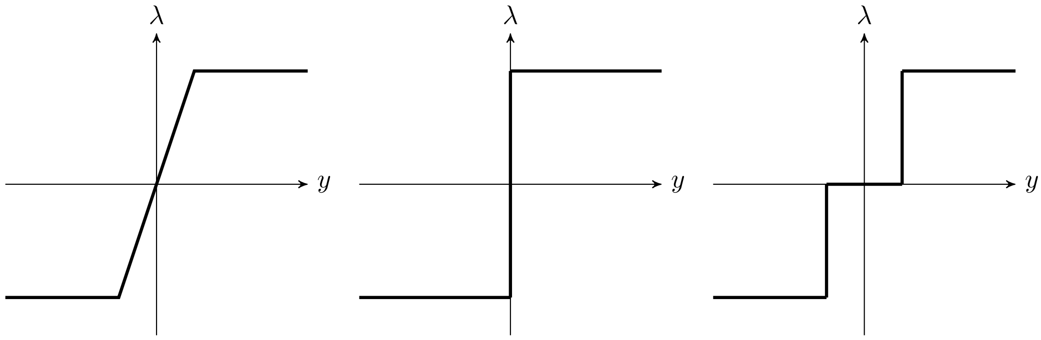
$$\mathrm{Sgn}(y) = \begin{cases} 1, & y > 0 \\ [-1, 1], & y = 0 \\ -1, & y < 0. \end{cases} \tag{18}$$

Using again convex analysis, the multivalued sign function may be formulated as an inclusion into a normal cone as

$$\lambda \in \mathrm{Sgn}(y) \Longleftrightarrow y \in N_{[-1,1]}(\lambda). \tag{19}$$

More generally, any system of the type,

$$\begin{cases} \dot{x}(t) = Ax(t) + a + B\lambda(t) \\ y(t) = Cx(t) + a \\ -\lambda(t) \in \mathrm{Sgn}(y(t)), \end{cases} \tag{20}$$



(a) Saturation (b) Relay (c) Relay with dead zone

Figure 2: Examples of multivalued piecewise linear models

can reformulated in terms of the following set-valued system

$$\begin{cases} \dot{x}(t) = Ax(t) + a + B\lambda(t) \\ y(t) = Cx(t) + c \\ -y(t) \in N_{[-1,1]^m}(\lambda(t)). \end{cases} \tag{21}$$

The system (21) appears in a lot of applications; among them, we can cite the sliding mode control, electrical circuits with relay and Zener diodes [4], or mechanical systems with friction [8].

Though this class of systems seems to be rather specific, it includes as well more general dynamical systems such as piecewise smooth systems and discontinuous ordinary differential equations. Indeed, the system (20) for scalars y and λ can be viewed as a discontinuous differential equation:

$$\dot{x}(t) = \begin{cases} Ax + a + B & \text{if} \quad Cx + c > 0 \\ Ax + a - B & \text{if} \quad Cx + c < 0. \end{cases} \tag{22}$$

One of the most well-known mathematical framework to deal with such systems is the Filippov theory [37] that embed the discontinuous differential equations into a differential inclusion. In the case of a single discontinuity surface given in our example by $S = \{x \mid Cx + c = 0\}$, the Filippov differential inclusion based on the convex hull of the vector fields in the neighborhood of S is equivalent to the use of the multivalued sign function in (20). Conversely, as it has been shown in [12], a piecewise smooth system can be formulated as a nonsmooth system based on products of multivalued sign functions.

1.2 Nonsmooth Dynamical systems in the large

Generally, the nonsmooth dynamical systems we propose to study mainly concern systems that possess the following features:

(i) A nonsmooth formulation of the constitutive/behavioral laws that define the system. Examples of nonsmooth formulations are piecewise smooth functions, multi–valued functions, inequality constraints, yielding various definitions of dynamical systems such as piecewise smooth systems, discontinuous ordinary differential equations, complementarity systems, projected dynamical systems, evolution or differential variational inequalities and differential inclusions (into normal cones). Fundamental mathematical tools come from convex analysis [63, 44, 43], complementarity theory [32], and variational inequalities theory [36].

(ii) A concept of solutions that does not require continuously differentiable functions of time. For instance, absolutely continuous, Lipschitz continuous functions or functions of local bounded variation are the basis for solution concepts. Measures or distributions are also solutions of interest for differential inclusions or evolution variational inequalities.

1.3 Nonsmooth systems versus hybrid systems

The nonsmooth dynamical systems we are dealing with, have a nonempty intersection with hybrid systems and cyber-physical systems, as it is briefly discussed in Sect. 3.1.4. Like in hybrid systems, nonsmooth dynamical systems define continuous–time dynamics that can be identified to modes separated by guards, defined by the constraints. However, the strong mathematical structure of nonsmooth dynamical systems allows us to state results on the following points:

(i) Mathematical concept of solutions: well-posedness (existence, and possibly, uniqueness properties, (dis)continuous dependence on initial conditions).

(ii) Dynamical systems theoretic properties: existence of invariants (equilibria, limit cycles, periodic solutions,. . .) and their stability, existence of oscillations, periodic and quasi-periodic solutions and propagation of waves.

(iii) Control theoretic properties: passivity, controllability, observability, stabilization, robustness.

These latter properties, that are common for smooth nonlinear dynamical systems, distinguish the nonsmooth dynamical systems from the very general definition of hybrid or cyber-physical systems [16, 42]. Indeed, it is difficult to give a precise mathematical concept of solutions for hybrid systems since the general definition of hybrid automata is usually too loose.

1.4 Numerical methods for nonsmooth dynamical systems

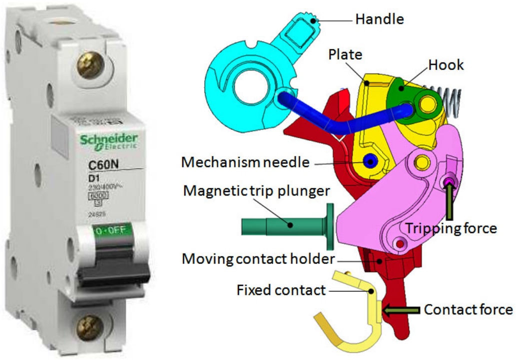
To conclude this brief exposition of nonsmooth dynamical systems, let us recall an important fact related to numerical methods. Beyond their intrinsic mathematical interest, and the fact that they model real physical systems, using nonsmooth dynamical systems as a model is interesting, because it exists a large set of robust and efficient numerical techniques to simulate them. Without entering into deeper details, let us give two examples of these techniques:

- *Numerical time integration methods: convergence, efficiency (order of consistency, stability, symplectic properties).* For the nonsmooth dynamical systems described above, there exist event–capturing time–stepping schemes with strong mathematical results. These schemes have the ability to numerically integrate the initial value problem without performing an event location, but by capturing the event within a time step. We call an event, or a transition, every change into the index set of the active constraints in the complementarity formulation or in the normal cone inclusion. Hence these schemes are able to simulate systems with a huge number of transitions or even worth finite accumulation of events (Zeno behavior). Furthermore, the schemes are not suffering from the weaknesses of the standard schemes based on a regularization (smoothing) of the multi-valued mapping resulting in stiff ordinary differential equations. For the time–integration of the initial value problem (IVP), or Cauchy problem, a lot of improvements of the standard time–stepping schemes for nonsmooth dynamics (Moreau–Jean time-stepping scheme) have been proposed in the last decade, in terms of accuracy and dissipation properties [1, 2, 64, 65, 3, 31, 27, 66, 29]. An important part of these schemes has been developed by members of the BIPOP team and has been implemented in the Siconos software (see Sect. 4.1).
- *Numerical solution procedure for the time–discretized problem, mainly through well-identified problems studied in the optimization and mathematical programming community.* Another very interesting feature is the fact that the discretized problem that we have to solve at each time–step is generally a well-known problem in optimization. For instance, for LCSs, we have to solve a linear complementarity problem [32] for which there exist efficient solvers in the literature. Comparing to the brute force algorithm with exponential complexity that consists in enumerating all the possible modes, the algorithms for linear complementarity problem have polynomial complexity when the problem is monotone.

In the Axis 2 of the research program (see Sect. 3.2), we propose to perform new research on the geometric time-integration schemes of nonsmooth dynamical systems, to develop new integration schemes for Boundary Value Problem (BVP), and to work on specific methods for two time-discretized problems: the Mathematical Program with Equilibrium Constraints (MPEC) for optimal control and Second Order Cone Complementarity Problems (SOCCP) for discrete frictional contact systems.

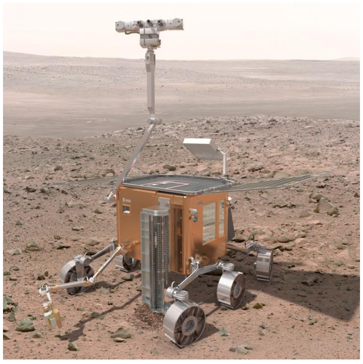
1.5 Application fields

Nonsmooth dynamical systems arise in a lot of application fields. We briefly expose here some applications that have been treated in the BIPOP team and that we will continue in the TRIPOP team, as a validation for the research axes and also in terms of transfer (Sect. 4.2). In mechanics, the main instances of nonsmooth dynamical systems are multibody systems with Signorini's unilateral contact, set-valued (Coulomb-like) friction and impacts, or in continuum mechanics, ideal plasticity, fracture or damage. Some illustrations are given in Figure 3(a-f). Other instances of nonsmooth dynamical systems can also be found in electrical circuits with ideal components (see Figure 3(g)) and in control theory, mainly with sliding mode control and variable structure systems (see Figure 3(h)). More generally, every time a piecewise, possibly set–valued, model of systems is invoked, we end up with a nonsmooth system. This is the case, for instance, for hybrid systems in nonlinear control or for piecewise linear modeling of gene regulatory networks in mathematical biology (see Figure 3(i)). Another common example of nonsmooth dynamics is also found when the vector field of a dynamical system is defined as a solution of an optimization problem under constraints, or a variational inequality. Examples of this kind are found in the optimal control theory, in dynamic Nash equilibrium or in the theory of dynamic flows over networks.

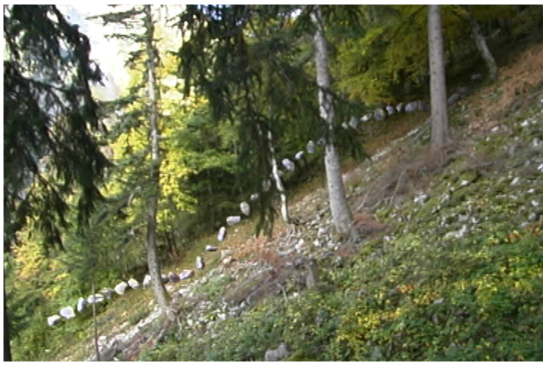


(a) Circuit breakers mechanisms [15]

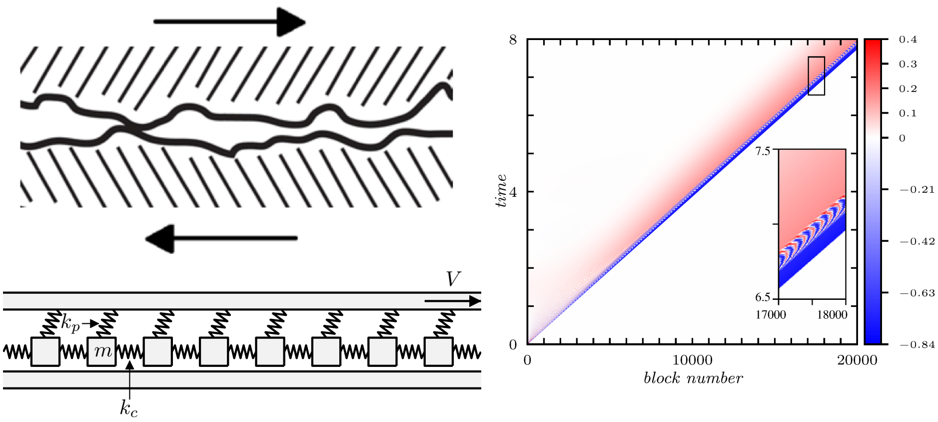
(b) Granular flows



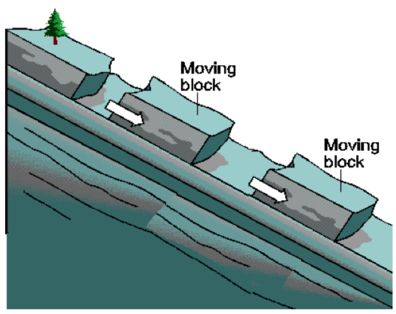
(c) Robots (ESA ExoMars Rover [6])



(d) Rockfall [21, 20, 35]

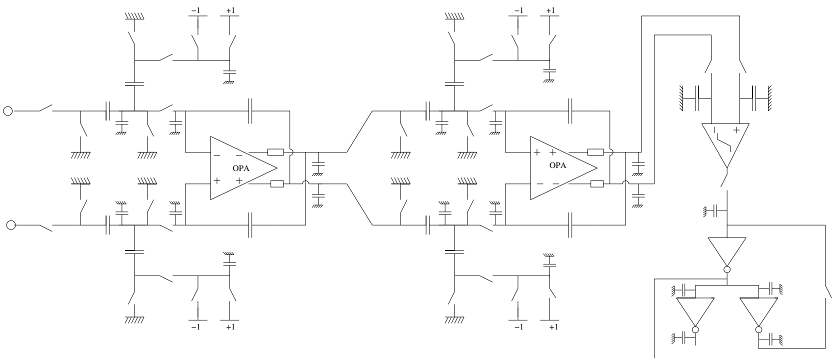


(e) Frictional interface and solitary waves in the Burridge-Knopoff model [57]

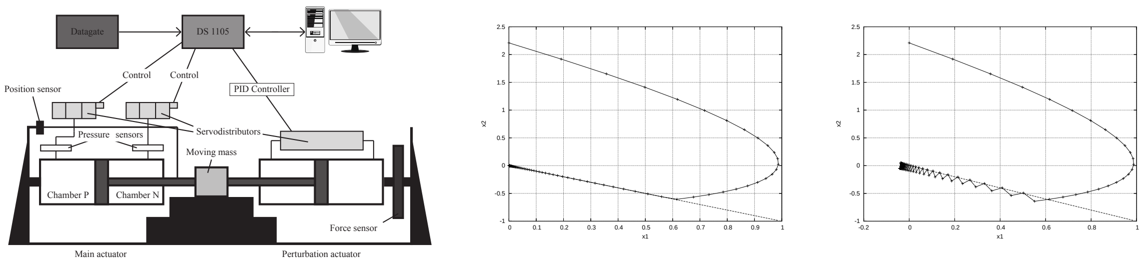


(f) Sliding blocks

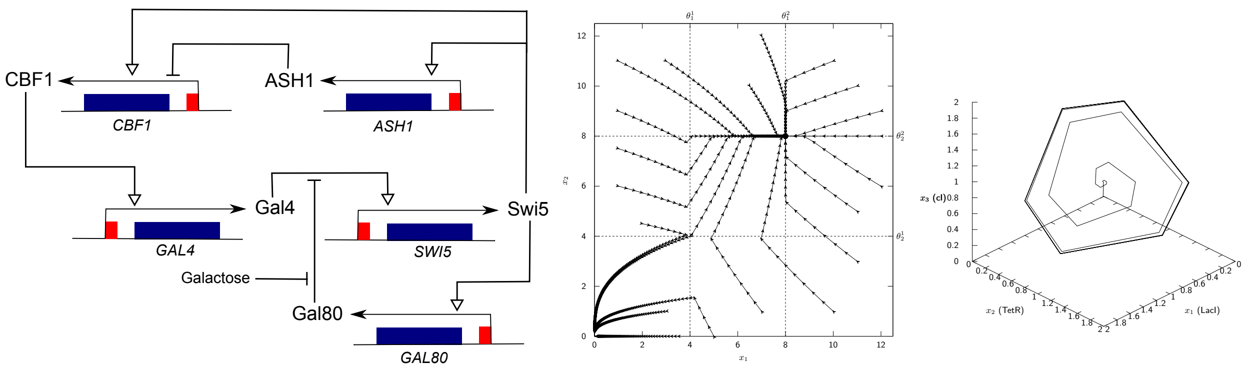
Figure 3: Application fields of nonsmooth dynamics (mechanics)



(g) Switched electrical circuits (delta-sigma converter) [4]



(h) Sliding mode control [9, 11, 46, 47, 56]



(i) Gene regulatory networks [12]

Figure 3: Application fields of nonsmooth dynamics (continued)

2 TRIPOP team

This section describes the composition of the team and its complementarity.

2.1 Members

The members of the team are:

- Research Scientists
  - Vincent Acary Inria, DR2 (LJK, HdR, Team Leader)
  - Franck Bourrier, IRSTEA[1], CR1 (LJK)
  - Bernard Brogliato, Inria, DR1 (LJK, HdR)
  - Arnaud Tonnelier, Inria, CRN (LJK)
- Faculty members
  - Guillaume James, Grenoble INP, Professor (LJK, HdR)
- External collaborator
  - Christophe Prieur, CNRS, DR1 (Gipsa-Lab UMR 5216, HdR)
- Post-doctoral fellows
  - Kirill Vorotnikov, Inria (2016–2018, G. James and B. Brogliato)
  - Achref El Mansour, Inria granted by STRMTG Grenoble[2] (co-supervision V. Acary, M. Weiss (STRMTG))
  - Post-doctoral fellow or starting research position granted by the FUI project Modeliscale coordinated by Dassault Systems to recruited in 2018.
- PhD students
  - Alexandre Vieira, Grenoble INP (2015–2018, B. Brogliato, C. Prieur)
  - Rami Sayoud, Schneider Electric, CIFRE grant (2018–2021, V. Acary, B. Brogliato)
  - Charlélie Bertrand granted by ENTPE (Ecole Nationale des Travaux Publics) (2018–2021) (co-supervision V. Acary, C. Lamarque (ENTPE))
  - PhD thesis, granted by Inria IPL Modeliscale to be recruited in 2018, (co-supervision V. Acary, B. Brogliato, B.Caillaud)
- Technical staff
  - Franck Pérignon, CNRS, IR2 (LJK, 20%)
  - Maurice Brémond, Inria, IR1 (LJK, SED, 40%)

2.2 Team members complementarity

The members of the proposal for TRIPOP possess different, complementary scientific skills and interests:

- Vincent Acary (Computational Mechanics, Control): graduated from the Ecole Centrale Marseille (Mechanical Engineering), Inria researcher since 2003. He got a PhD in 2001 in Mechanics from the University Aix–Marseille II and an HdR in 2015 from the Grenoble University (doctoral school EDMSTII). His current research interests are the modeling and the numerical methods for nonsmooth dynamical systems and the sliding-mode control. He is the main designer of the SICONOS software enabling industrial applications towards multibody systems with contact and Coulomb friction. He has supervised 4 PhD and 4 post-doc students.
- Franck Bourrier (Impact Mechanics, granular material, geomechanics, natural hazards) graduated from INSA Lyon (Mechanics) in 2002 and got a PhD in Mechanics from INP Grenoble in 2008. He is researcher in Irstea Grenoble since 2009. His research focuses on the modeling of the effect of forests on natural hazards, in particular rockfall. This applied research field is closely related with the modeling of nonsmooth dynamical systems in the context of complex materials and geometry. He participated in the design of numerous block propagation models used by practitioners and to

[1] http://www.irstea.fr/linstitut/nos-centres/grenoble

[2] Service Technique des Remontées Mécaniques et des Transports Guidés, http://www.strmtg.developpement-durable.gouv.fr/

several expertises in the field of rockfall hazard mitigation. He supervised 3 PhD and 3 post-doc students.

- Bernard Brogliato (Control, Impact Mechanics, Dissipative Systems): graduated from the Ecole Normale Supérieure de Cachan (Mechanical Engineering), got a PhD and an HdR from the Grenoble Institute of Technology in Control, in 1991 and 1995 respectively. He is Senior Researcher at Inria since 2001, and was founder and leader of the BIPOP team. His current research interests are in impact mechanics, control and state observation of nonsmooth mechanical systems, and a little bit of mathematical analysis for nonsmooth systems. He has supervised and co-supervised 15 PhD and 9 post-doc students.
- Guillaume James (Theoretical and applied nonlinear waves): graduated from the University of Nice - Sophia Antipolis, MSc (1996), got a PhD Univ. Nice - Sophia Antipolis (1999) and an Habilitation Thesis Univ. Paul Sabatier Toulouse 3 (2005). His research concerns the mathematical modeling and analysis of nonlinear wave phenomena. His current research work applies to granular metamaterials (compression waves, vibrations) and frictional systems (stick-slip). His research focuses mainly on the occurrence of solitary waves (localized traveling waves) and breathers (localized oscillations) in nonlinear media, with an emphasis on the effect of spatial discreteness. Understanding these phenomena requires to address a variety of infinite-dimensional dynamical systems ranging from lattice differential equations and advance-delay equations to PDEs.
- Christophe Prieur (Control, Nonlinear Dynamics, infinite–dimensional systems, PDE): graduated from the Ecole Normale Supérieure de Cachan (Mathematics), got a PhD from the Université Paris-Sud in Applied Mathematics in 2001 and an HdR from Université de Toulouse in 2009. He is a CNRS Senior Researcher at Gipsa-lab, Grenoble. His current research interests include nonlinear control theory (for finite and infinite-dimensional systems), and control of partial differential equations.
- Arnaud Tonnelier (Nonlinear Dynamics, Mechanical waves, Excitable systems): graduated from ENSIMAG, is an Inria researcher in Applied Mathematics since 2003. He got a PhD in applied mathematics from the Joseph Fourier University in 2001. He was a researcher at the Loria (Nancy) and he joined the Inria Grenoble research centre in 2006. His research interests are in the applications of nonlinear dynamical systems and, in particular, in the study of neural systems and excitable mechanical systems. He has supervised 3 post-doc and one PhD students.

To complete the description of Team members complementarity, we give in Figure 4 a graph of PhD and Post-doc supervisions.

2.3 Recruitments

The Tripop team is aware about the fact that we need to increase our supervision rate in terms of PhD students and post-doctoral fellows. In this perspective, we will propose the following PhD subjects to calls for PhD grants:

- PhD thesis on control of linear complementarity systems (B. Brogliato, C. Prieur)
- PhD thesis on numerical optimization for discrete frictional contact problems (V. Acary, P. Armand)
- PhD thesis on non-smooth modeling and simulation of energy dissipation processes during rockfall (V. Acary, F. Bourrier)

We will also continue discussion with industrial partners on the following projects:

- PhD thesis CIFRE grants with the companies POMA[3], IMSRN[4] in discussion for 2018
- Post-doctoral fellow or CIFRE PhD granted by ANSYS France in discussion for 2018.

For the permanent staff, our link with the LJK laboratory would be reinforced by a recruitment of an associate professor ("Maître de Conférences"). For this purpose, a proposition of position has been made at the interface between EDP team and TRIPOP team, on numerical modeling in geosciences. Concerning an INRIA permanent researcher, an excellent candidate in one of the research themes of the project would be very valuable. An emphasis on a numerical method and high performance scientific computing in view of our new industrial applications, will be encouraged.

[3] http://www.poma.net/

[4] Ingénierie des Mouvements de Sol et des Risques Naturels, http://www.imsrn.com/en/

3 Scientific objectives

In this section, we develop our scientific program. In the framework of nonsmooth dynamical systems, the activities of the project–team will be on focused on the following research axes:

- Axis 1: Modeling and analysis (detailed in Sect. 3.1).
- Axis 2: Numerical methods and simulation (detailed in Sect. 3.2).
- Axis 3: Automatic Control (detailed in Sect. 3.3)

These research axes will be developed with a strong emphasis on the software development and the industrial transfer that are detailed respectively in Sect. 4.1 and Sect. 4.2.

Timeline and priorities The research program detailed in this section is quite dense and all the scientific objectives will not be started at the beginning of the project. Most of these scientific objectives are also subjected to the recruitment and the financial support. This aspect will also affect the way we start and develop out scientific program. Nevertheless, we specify for each action the subjects that we could start right at the beginning of the project (short-term actions, within 4 years) and a list of subjects that are on a longer term (4-8 years).

3.1 Axis 1: Modeling and analysis

This axis is dedicated to the modeling and the mathematical analysis of nonsmooth dynamical systems. It consists of four main directions. Two directions are in the continuation of BIPOP activities: 1) multibody vibro-impact systems (Sect. 3.1.1) and 2) excitable systems (Sect. 3.1.2). Two directions are completely new with respect to BIPOP: 3) Nonsmooth geomechanics and natural hazards assessment (Sect. 3.1.3) and 4) Cyber-physical systems (hybrid systems) (Sect. 3.1.4).

3.1.1 Multibody vibro-impact systems

Participants: B. Brogliato, F. Bourrier, G. James, V. Acary

- *Multiple impacts with or without friction (short-term):* there are many different approaches to model collisions, especially simultaneous impacts (so-called multiple impacts)[60]. One of our objectives is on one hand to determine the range of application of the models (for instance, when can one use "simplified" rigid contact models relying on kinematic, kinetic or energetic coefficients of restitution?) on typical benchmark examples (chains of aligned beads, rocking block systems). On the other hand, try to take advantage of the new results on nonlinear waves phenomena, to better understand multiple impacts in 2D and 3D granular systems. The study of multiple impacts with (unilateral) nonlinear visco-elastic models (Simon-Hunt-Crossley, Kuwabara-Kono), or visco-elasto-plastic models (assemblies of springs, dashpots and dry friction elements), is also a topic of interest, since these models are widely used.
- *Artificial or manufactured or ordered granular crystals, meta-materials (short-term):* Granular metamaterials (or more general nonlinear mechanical metamaterials) offer many perspectives for the passive control of waves originating from impacts or vibrations. The analysis of waves in such systems is delicate due to spatial discreteness, nonlinearity and non-smoothness of contact laws [62, 48, 49, 55]. We will use a variety of approaches, both theoretical (e.g. bifurcation theory, modulation equations) and numerical, in order to describe nonlinear waves in such systems, with special emphasis on energy localization phenomena (excitation of solitary waves, fronts, breathers).
- *Systems with clearances, modeling of friction (long-term):* joint clearances in kinematic chains deserve specific analysis, especially concerning friction modeling[15]. Indeed contacts in joints are often conformal, which involve large contact surfaces between bodies. Lubrication models should also be investigated.
- *Painlevé paradoxes (long-term):* the goal is to extend the results in [39], which deal with single-contact systems, to multi-contact systems. One central difficulty here is the understanding and the analysis of singularities that may occur in sliding regimes of motion.

As a continuation of the work in the BIPOP team, our software code, Siconos (see Sect. 4.1) will be our favorite software platform for the integration of these new modeling results.

3.1.2 Excitable systems (short-term)

Participants: A. Tonnelier, G. James

An excitable system elicits a strong response when the applied perturbation is greater than a threshold [57, 58, 19, 67]. This property has been clearly identified in numerous natural and physical systems. In mechanical systems, non-monotonic friction law (of spinodal-type) leads to excitability. Similar behavior may be found in electrical systems such as active compounds of neuristor type. Models of excitable systems incorporate strong non-linearities that can be captured by non-smooth dynamical systems. Two properties are deeply associated with excitable systems: oscillations and propagation of nonlinear waves (autowaves in coupled excitable systems). We aim at understanding these two dynamical states in excitable systems through theoretical analysis and numerical simulations. Specifically we plan to study:

- Threshold-like models in biology: spiking neurons, gene networks.
- Frictional contact oscillators (slider block, Burridge-Knopoff model).
- Dynamics of active electrical devices : memristors, neuristors.

3.1.3 Nonsmooth geomechanics and natural hazards assessment

Participants: F. Bourrier, B. Brogliato, G. James, V. Acary

- *Rockfall impact modeling (short-term):* Trajectory analysis of falling rocks during rockfall events is limited by a rough modeling of the impact phase [21, 20, 54]. The goal of this work is to better understand the link between local impact laws at contact with refined geometries and the efficient impact laws written for a point mass with a full reset map. A continuum of models in terms of accuracy and complexity will be also developed for the trajectory studies. In particular, nonsmooth models of rolling friction, or rolling resistance will be developed and formulated using optimization problems.
- *Experimental validation (short-term):* The participation of IRSTEA with F. Bourrier makes possible the experimental validation of models and simulations through comparisons with real data. IRSTEA has a large experience of lab and in-situ experiments for rockfall trajectories modeling [21, 20]. It is a unique opportunity to strengthen our model and to prove that nonsmooth modeling of impacts is reliable for such experiments and forecast of natural hazards.
- *Rock fracturing (long-term):* When a rock falls from a steep cliff, it stores a large amount of kinetic energy that is partly dissipated though the impact with the ground. If the ground is composed of rocks and the kinetic energy is sufficiently high, the probability of the fracture of the rock is high and yields an extra amount of dissipated energy but also an increase of the number of blocks that fall. In this item, we want to use the capability of the nonsmooth dynamical framework for modeling cohesion and fracture [50, 13] to propose new impact models.
- *Rock/forest interaction (long-term):* To prevent damages and incidents to infrastructures, a smart use of the forest is one of the ways to control trajectories (decrease of the run-out distance, jump heights and the energy) of the rocks that fall under gravity [34, 35]. From the modeling point of view and to be able to improve the protective function of the forest, an accurate modeling of impacts between rocks and trees is required. Due to the aspect ratio of the trees, they must be considered as flexible bodies that may be damaged by the impact. This new aspect offers interesting modeling research perspectives.

More generally, our collaboration with IRSTEA opens new long term perspectives on granular flows applications such as debris and mud flows, granular avalanches and the design of structural protections. The numerical methods that go with these new modeling approaches will be implemented in our software code, Siconos (see Sect. 4.1)

3.1.4 Cyber-physical systems (hybrid systems)

Participants: V. Acary, B. Brogliato, C. Prieur, A. Tonnelier

Nonsmooth systems have a non-empty intersection with hybrid systems and cyber–physical systems. However, nonsmooth systems enjoy strong mathematical properties (concept of solutions, existence and uniqueness) and efficient numerical tools. This is often the result of the fact that nonsmooth dynamical systems are models of physical systems, and then, take advantage of their intrinsic property (conservation or dissipation of energy, passivity, stability). A standard example is a circuit with n ideal diodes. From the hybrid point of view, this circuit is a piecewise smooth dynamical system with 2^n modes, that can be quite cumbersome to enumerate in order to determinate the current mode. As a nonsmooth system, this circuit can be formulated as a complementarity system for which there exist efficient time–stepping schemes and polynomial time algorithms for the computation of the current mode. The key idea of this research action is to take benefit of this observation to improve the hybrid system modeling tools.

Research actions: There are two main actions in this research direction that will be implemented in the framework of the Inria Project Lab (IPL " Modeliscale", see https://team.inria.fr/modeliscale/ for partners and details of the research program):

- *Structural analysis of multimode DAE (short-term):* When a hybrid system is described by a Differential Algebraic Equation (DAE) with different differential indices in each continuous mode, the structural analysis has to be completely rethought. In particular, the re-initialization rule, when a switching occurs from a mode to another one, has to be consistently designed. We propose in this action to use our knowledge in complementarity and (distribution) differential inclusions [10] to design consistent re-initialization rule for systems with nonuniform relative degree vector $(r_1, r_2, \dots, r_m)$ and $r_i \neq r_j, i \neq j$.
- *Cyber–physical in hybrid systems modeling languages (short-term):* Nowadays, some hybrid modeling languages and tools are widely used to describe and to simulate hybrid systems (MODELICA, SIMULINK, and see [30] for references therein). Nevertheless, the compilers and the simulation engines behind these languages and tools suffer from several serious weaknesses (failure, weird output or huge sensitivity to simulation parameters), especially when some components, that are standard in nonsmooth dynamics, are introduced (piecewise smooth characteristic, unilateral constraints and complementarity condition, relay characteristic, saturation, dead zone, . . .). One of the main reasons is the fact that most of the compilers reduce the hybrid system to a set of smooth modes modeled by differential algebraic equations and some guards and reinitialization rules between these modes. Sliding mode and Zeno–behaviour are really harsh for hybrid systems and relatively simple for nonsmooth systems. With B. Caillaud (Inria HYCOMES) and M. Pouzet (Inria PARKAS), we propose to improve this situation by implementing a module able to identify/describe nonsmooth elements and to efficiently handle them with SICONOS as the simulation engine. They have already carried out a first implementation [28] in Zelus, a synchronous language for hybrid systems http://zelus.di.ens.fr. Removing the weaknesses related to the nonsmoothness of solutions should improve hybrid systems towards robustness and certification.
- *A general solver for piecewise smooth systems (long-term)* This direction is the continuation of the promising result on modeling and the simulation of piecewise smooth systems [12]. As for general hybrid automata, the notion or concept of solutions is not rigorously defined from the mathematical point of view. For piecewise smooth systems, multiplicity of solutions can happen and sliding solutions are common. The objective is to recast general piecewise smooth systems in the framework of differential inclusions with Aizerman–Pyatnitskii extension [12, 37]. This operation provides a precise meaning to the concept of solutions. Starting from this point, the goal is to design and study an efficient numerical solver (time–integration scheme and optimization solver) based on an equivalent formulation as mixed complementarity systems of differential variational inequalities. We are currently discussing the issues in the mathematical analysis. The goal is to prove the convergence of the time–stepping scheme to get an existence theorem. With this work, we should also be able to discuss the general Lyapunov stability of stationary points of piecewise smooth systems.

3.2 Axis 2: Numerical methods and simulation

This axis is dedicated to the numerical methods and simulation for nonsmooth dynamical systems. As we mentioned in the introduction, the standard numerical methods have been largely improved in terms of accuracy and dissipation properties in the last decade. Nevertheless, the question of the geometric time–integration techniques remains largely open. It constitutes the objective of the first research direction in Sect. 3.2.1. Beside the standard IVP, the question of normal mode analysis for nonsmooth systems is also a research topic that emerged in the recent years. More generally, the goal of the second research direction (Sect. 3.2.2) is to develop numerical methods to solve boundary value problems in the nonsmooth framework. This will serve as a basis for the computation of the stability and numerical continuation of invariants. Finally, once the time-integration method is chosen, it remains to solve the one-step nonsmooth problem, which is, most of time, a numerical optimization problem. In Sect. 3.2.3, we propose to study two specific problems with a lot of applications: the

Mathematical Program with Equilibrium Constraints (MPEC) for optimal control, and Second Order Cone Complementarity Problems (SOCCP) for discrete frictional contact systems. After some possible prototypes in scripting languages (Python and Matlab), we will be attentive that all these developments of numerical methods will be integrated in Siconos.

3.2.1 Geometric time–integration schemes for nonsmooth Initial Value Problem (IVP) (short-term)

Participants: V. Acary, B. Brogliato, G. James, F. Pérignon
The objective of this research item is to continue to improve classical time–stepping schemes for nonsmooth systems to ensure some qualitative properties in discrete-time. In particular, the following points will be developed

- Conservative and dissipative systems. The question of the energy conservation and the preservation of dissipativity properties in the Willems sense [41] will be pursued and extended to new kinds of systems (nonlinear mechanical systems with nonlinear potential energy, systems with limited differentiability (rigid impacts vs. compliant models)).
- Lie–group integration schemes for finite rotations for the multi-body systems extending recent progresses in that directions for smooth systems [17].
- Conservation and preservation of the dispersion properties of the (non)-dispersive system.

3.2.2 Stability and numerical continuation of invariants

Participants: G. James, V. Acary, A. Tonnelier, F. Pérignon,

By invariants, we mean equilibria, periodic solutions, limit cycles or waves. Our preliminary work on this subject raised the following research perspectives:

- Computation of periodic solutions of discrete mechanical systems (short-term). The modal analysis, *i.e.*, a spectral decomposition of the problem into linear normal modes is one of the basic tools for mechanical engineers to study dynamic response and resonance phenomena of an elastic structure. Since several years, the concept of nonlinear normal modes [51], that is closely related to the computation of quasi-periodic solutions that live in a nonlinear manifold, has emerged as the nonlinear extension of the modal analysis. One of the fundamental question is: what remains valid if we add unilateral contact conditions ? The computation of nonsmooth modes amounts to computing periodic solutions, performing the parametric continuation of solution branches and studying the stability of these branches. This calls for time integration schemes for IVP an BVP that satisfy some geometric criteria: conservation of energy, reduced numerical dispersion, symplecticity as we described before. Though the question of conservation of energy for unilateral contact has been discussed in [3], the other questions remain open. For the shooting technique and the study of stability, we need to compute the Jacobian matrix of the flow with respect to initial conditions, the so-called saltation matrix [52, 61] for nonsmooth flows. The eigenvalues of this matrix are the Floquet multipliers that give some information on the stability of the periodic solutions. The question of an efficient computation of this matrix is also an open question. For the continuation, the question is also largely open since the continuity of the solutions with respect to the parameters is not ensured.
- Extension to elastic continuum media (long-term). This is a difficult task. First of all, the question of the mathematical model for the dynamic continuum problem with unilateral contact raises some problems of well–posedness. For instance, the need for an impact law is not clear in some cases. If we perform a semi–discretization in space with classical techniques (Finite Element Methods, Finite Difference Schemes), we obtain a discrete system for which the impact law is needed. Besides all the difficulties that we enumerate for discrete systems in the previous paragraph, the space discretization also induces numerical dispersion that may destroy the periodic solutions or renders their computation difficult. The main targeted applications for this research are cable–systems, string musical instruments, and seismic response of electrical circuit breakers with Schneider Electric.
- Computation of solutions of nonsmooth time Boundary Value Problems (BVP) (collocation, shooting) (long-term). The technique developed in the two previous items can serve as a basis for the development of more general solvers for nonsmooth BVP that can be for instance found when

we solve optimal control problems by direct or indirect methods, or the computation of nonlinear waves. Two directions can be envisaged:

  - Shooting and multiple shooting techniques. In such methods, we reformulate the BVP into a sequence of IVPs that are iterated through a Newton based technique. This implies the computation of Jacobians for nonsmooth flows, the question of the continuity w.r.t to initial condition and the use of semi-smooth Newton methods.
  - Finite differences and collocations techniques. In such methods, the discretization will result into a large sparse optimization problems to solve. The open questions are as follows: a) the study of convergence, b) how to locally improve the order if the solution is locally smooth, and c) how to take benefit of spectral methods.

- Continuation techniques of solutions with respect to a parameter. Standard continuation technique requires smoothness. What types of methods can be extended in the nonsmooth case (arc-length technique, nonsmooth (semi-smooth) Newton, Asymptotical Numerical Methods (ANM))

3.2.3 Numerical optimization for discrete nonsmooth problems

Participants: V. Acary, M. Brémond, F. Pérignon, B. Brogliato, C. Prieur

- Mathematical Program with Equilibrium Constraints (MPEC) for optimal control (long-term). The discrete problem that arises in nonsmooth optimal control is generally a MPEC [68]. This problem is intrinsically nonconvex and potentially nonsmooth. Its study from a theoretical point of view has started 10 years ago but there is no consensus for its numerical solving. The goal is to work with world experts of this problem (in particular M. Ferris from Wisconsin University) to develop dedicated algorithms for solving MPEC, and provide to the optimization community challenging problems.
- Second Order Cone Complementarity Problems (SOCCP) for discrete frictional systems (short-term): After some extensive comparisons of existing solvers on a large collection of examples [7, 5], the numerical treatment of constraints redundancy by the proximal point technique and the augmented Lagrangian formulation seems to be a promising path for designing new methods. From the comparison results, it appears that the redundancy of constraints prevents the use of second order methods such as semi–smooth Newton methods or interior point methods. With P. Armand (XLIM, U. de Limoges), we propose to adapt recent advances for regularizing constraints for the quadratic problem [38] for the second-order cone complementarity problem. The other question is the improvement of the efficiency of the algorithms by using accelerated schemes for the proximal gradient method that come from large–scale machine learning and image processing problems. Learning from the experience in large–scale machine learning and image processing problems, the accelerated version of the classical gradient algorithm [59] and the proximal point algorithm [18], and many of their further extensions, could be of interest for solving discrete frictional contact problems. Following the visit of Y. Kanno (University of Tokyo) and his preliminary experience on frictionless problems, we will extend its use to frictional contact problem. When we face large-scale problems, the main available solvers is based on a Gauss–Seidel strategy that is intrinsically sequential. Accelerated first-order methods could be a good alternative to take benefit of the distributed scientific computing architectures.

3.3 Axis 3: Automatic Control

Participants: B. Brogliato, C. Prieur, V. Acary

This last axis is dedicated to the automatic control of nonsmooth dynamical systems, or the nonsmooth control of smooth systems. The first item concerns the discrete-time sliding mode control for which significant results on the implicit implementation have been obtained in the BIPOP team. The idea is to pursue this research towards state observers and differentiators (Sect 3.3.1). The second direction concerns the optimal control which brings of nonsmoothness in their solution and their formulation. After the preliminary work in BIPOP on the quadratic optimal control of Linear Complementarity systems(LCS), we propose to go further to the minimal time problem, to impacting systems and optimal control with state constraints (Sect. 3.3.2). In Sect 3.3.3, the objective is to study the control of nonsmooth systems that contain unilateral constraint, impact and friction. The targeted systems are cable–driven systems, multi-body systems with clearances and granular materials. In Sect 3.3.4, we will continue our work on the higher order Moreau sweeping process. Up to now, the work of BIPOP was restricted to finite-dimensional systems. In Sect 3.3.5, we propose to extend our approach to the control of elastic structures subjected to contact unilateral constraints.

It is noteworthy that most of the problems listed below, will make strong use of the numerical tools analyzed in Axis 2, and of the Modeling analysis of Axis 1. For instance all optimal control problems yield BVPs. Control of granular materials will undoubtedly use models and numerical simulation developed in Axis 1 and 2. And so on. It has to be stressed that the type of nonsmooth models we are working with, deserve specific numerical algorithms which cannot be found in commercial software packages. One of the goals is to continue to extend our software package Siconos, and in particular the siconos/control toolbox with these developments.

3.3.1 Discrete-time Sliding-Mode Control (SMC) and State Observers (SMSO) (short-term)

- *SMSO, exact differentiators:* we have introduced and obtained significant results on the implicit discretization of various classes of sliding-mode controllers [9, 11, 45, 56, 24], with successful experimental validations [46, 45, 47, 69]. Our objective is to prove that the implicit discretization can also bring advantages for sliding-mode state observers and Levant's exact differentiators, compared with the usual explicit digital implementation that generates chattering. In particular the implicit discretization guarantees Lyapunov stability and finite-time convergence properties which are absent in explicit methods.
- *High-Order SMC (HOSMC):* this family of controllers has become quite popular in the sliding-mode scientific community since its introduction by Levant in the nineties. We want here to continue the study of implicit discretization of HOSMC (twisting, super-twisting algorithms) and especially we would like to investigate the comparisons between classical (first order) SMC and HOSMC, when both are implicitly discretized, in terms of performance, accuracy, chattering suppression. Another topic of interest is stabilization in finite-time of systems with impacts and unilateral constraints, in a discrete-time setting.

3.3.2 Optimal Control

- *Linear Complementarity Systems (LCS) (short-term):* With the PhD thesis of A. Vieira, we have started to study the quadratic optimal control of LCS. Our objective is to go further with minimum-time problems. Applications of LCS are mainly in electrical circuits with set-valued components such as ideal diodes, transistors, *etc.* Such problems naturally yield MPEC when numerical solvers are sought. It is therefore intimately linked with Axis 2 objectives.
- *Impacting systems (long-term):* the optimal control of mechanical systems with unilateral constraints and impacts, largely remains an open issue. The problem can be tackled from various approaches: vibro-impact systems (no persistent contact modes) that may be transformed into discrete-time mappings *via* the impact Poincaré map; or the classical integral action minimization (Bolza problem) subjected to the complementarity Lagrangian dynamics including impacts.
- *State constraints, generalized control (long-term):* this problem differs from the previous two, since it yields Pontryagin's first order necessary conditions that take the form of an LCS with higher relative degree between the complementarity variables. This is related to the numerical techniques for the higher order sweeping process [10].

3.3.3 Control of nonsmooth discrete Lagrangian systems (short-term)

- *Cable–driven systems:* these systems are typically different from the cable-car systems, and are closer in their mechanical structure to so-called tensegrity structures. The objective is to actuate a system *via* cables supposed in a first instance to be flexible (slack mode) but non-extensible in their longitudinal direction. This gives rise to complementarity conditions, one big difference with usual complementarity Lagrangian systems being that the control actions operate directly in one of the complementary variables (and not in the smooth dynamics as in cable-car systems). Therefore both the cable models and the control properties are expected to differ a lot from what we may use for cableway systems (for which guaranteeing a positive cable tension is usually not an issue, hence avoiding slack modes, but the deformation of the cables due to the nacelles and cables weights, is an important factor). Tethered systems are a close topic.

- *Multi-body systems with clearances:* our approach is to use models of clearances with dynamical impact effects, *i.e.* within Lagrangian complementarity systems. Such systems are strongly underactuated due to mechanical play at the joints. However their structure, as underactuated systems, is quite different from what has been usually considered in the Robotics and Control literature. In the recent past we have proposed a thorough numerical robustness analysis of various feedback collocated and non-collocated controllers (PD, linearization, passivity-based). We propose here to investigate specific control strategies tailored to such underactuated systems [23].
- *Granular systems:* the context is the feedback control of granular materials. To fix the ideas, one may think of a "juggling" system whose "object" (uncontrolled) part consists of a chain of aligned beads. Once the modeling step has been fixed (choice of a suitable multiple impact law), one has to determine the output to be controlled: all the beads, some of the beads, the chain's center of mass (position, velocity, vibrational magnitude and frequency), *etc.* Then we aim at investigating which type of controller may be used (output or state feedback, "classical" or sinusoidal input with feedback through the magnitude and frequency) and especially which variables may be measured/observed (positions and/or velocities of all or some of the beads, position and/or velocity of the chain's center of gravity). This topic follows previous results we obtained on the control of juggling systems [25], with increasing complexity of the "object"'s dynamics. The next step would be to extend to 2D and then 3D granular materials. Applications concern vibrators, screening, transport in mining and manufacturing processes.
- *Stability of structures:* our objective here is to study the stability of stacked blocks in 2D or 3D, and the influence on the observed behavior (numerically and/or analytically) of the contact/impact model.

3.3.4 Switching LCS and DAEs, higher-order sweeping process (HOSwP) (short-term)

- We have gained a strong experience in the field of complementarity systems and distribution differential inclusions [10, 26], that may be seen as some kind of switching DAEs. We plan to go further with non-autonomous HOSwP with switching feedback inputs and non-uniform vector relative degrees. Switching linear complementarity systems can also be studied, though the exact relationships between both point of views remain unclear at the present time. This axis of research is closely related to cyber-physical systems in section 3.1.

3.3.5 Control of Elastic (Visco-plastic) systems with contact, impact and friction (short-term)

- *Stabilization, trajectory tracking:* until now we have focused on the stability and the feedback control of systems of rigid bodies. The proposal here is to study the stabilization of flexible systems (for instance, a "simple" beam) subjected to unilateral contacts with or without set-valued friction (contacts with obstacles, or impacts with external objects line particle/beam impacts). This gives rise to varying (in time and space) boundary conditions. The best choice of a good contact law is a hard topic discussed in the literature.
- *Cableway systems (STRMTG, POMA):* cable-car systems present challenging control problems because they usually are underactuated systems, with large flexibilities and deformations. Simplified models of cables should be used (Ritz-Galerkin approach), and two main classes of systems may be considered: those with moving cable and only actuator at the station, and those with fixed cable but actuated nacelles. It is expected that they possess quite different control properties and thus deserve separate studies. The nonsmoothness arises mainly from the passage of the nacelles on the pylons, which induces frictional effects and impacts. It may certainly be considered as a nonsmooth set-valued disturbance within the overall control problem.

3.4 Scientific novelty and new applications

In this section, we discussed briefly our positioning with respect to the BIPOP team.

New composition of the team Four members of BIPOP will not participate in TRIPOP: C. Lemaréchal (retired in April 2014), J. Malick (left in January 2016, now in charge of team DAO at LJK), P.B. Wieber (plans to launch a new team on biped robotics), F. Bertails-Descoubes (now in charge of the team ELAN at Inria). G. James joined BIPOP four years ago. F. Bourrier has joined the team to complete our experience in impact mechanics, geophysical natural hazards and real-case applications on rockfall risks.

Novelty in the scientific contents BIPOP was originally constructed around nonsmooth optimization and contact mechanics. Activities linked with biped Robotics, theoretical Optimization (combinatorial, algorithms for machine learning), and applications to Computer Graphics, will no longer exist in TRIPOP. The new team TRIPOP will be focused on nonsmooth dynamical systems, with strong emphasis on mechanics and control. Though these scientific topics were already present in BIPOP, we plan to launch many completely new activities among them

- hybrid and cyber-physical systems (Sect. 3.1.4),
- nonsmooth geomechanics (Sect. 3.1.3)
- excitable systems and nonlinear waves in discrete media (Sect 3.1.1 and 3.1.2),
- nonsmooth dynamics and normal modes of flexible systems (Sect 3.1.1 and 3.2.2),
- optimal control of nonsmooth dynamical systems (Sect. 3.3.2),
- control of cable-driven robots (Sect. 3.3.5 and 3.3.3) and granular material (Sect. 3.3.3),

with completely new applications and domains

- natural hazards modeling (Sect. 3.1.3 and 4.2),
- cable transport systems (Sect. 3.1.1 and 4.2),
- circuits with memristors and neuristors(Sect. 3.1.2).

3.5 Collaboration within the team via co-supervision of PhD students and postdoctoral fellows

Here we list of PhD subjects which could rise to co-supervisions in the team and we give a graph representation of this collaboration in Figure 4. When the PhD and Post-doc will be granted by governmental funds (Inria CORDI, Doctoral schools), the priority will be given to co-supervised subjects [5].

PhD 1 *Influence of the vibrational environment on the functional conditions of circuit breakers* CIFRE granted by Schneider Electric co-supervised by V. Acary and B. Brogliato

PhD 2 *Trajectory tracking for complementary systems* co-supervised by B. Brogliato and C. Prieur.

PhD 3 *Modeling and control of flexible structures with unilateral contact and friction* co-supervised by B. Brogliato and C. Prieur.

PhD 4 *Modeling and control of cable-driven systems* co-supervised by B. Brogliato and C. Prieur.

PhD 5 *Robust numerical methods for block trajectory simulations integrating block fragmentation* co-supervised by V. Acary, F. Bourrier, and B. Brogliato.

PhD 6 *Painlevé paradoxes in multibody systems with friction* co-supervised by B. Brogliato and G. James.

PhD 7 *Excitable mechanical systems with contact and friction* co-supervised by G. James and A. Tonnelier.

PhD 8 *Simplified impact model for the wave-propagation in granular media* co-supervised by B. Brogliato and G. James.

PhD 9 *Numerical modeling of rockfall fence and avalanche prevention protection systems* co-supervised by V. Acary and F. Bourrier.

PhD 10 *Control of systems with clearances* co-supervised by V. Acary and B. Brogliato.

[5]The name of the advisors are given in alphabetical order

PhD 11 *Implicit discrete time observers and differentiators* co-supervised by V. Acary and B. Brogliato.

PhD 12 *Structural analysis of multi-mode LCS* co-supervised by V. Acary and B. Brogliato.

PhD 12 *Optimal control of impacting systems LCS* co-supervised by V. Acary, B. Brogliato. and C. Prieur

Post-doc 1 *Numerical modeling of the dynamics cable transport systems* Post doctoral fellow granted by STRMTG co-supervised by V. Acary and B. Brogliato

Post-doc 2 *Modeling and analysis of electrical circuits with neuristors and memresitors* co-supervised by V. Acary and A. Tonnelier.

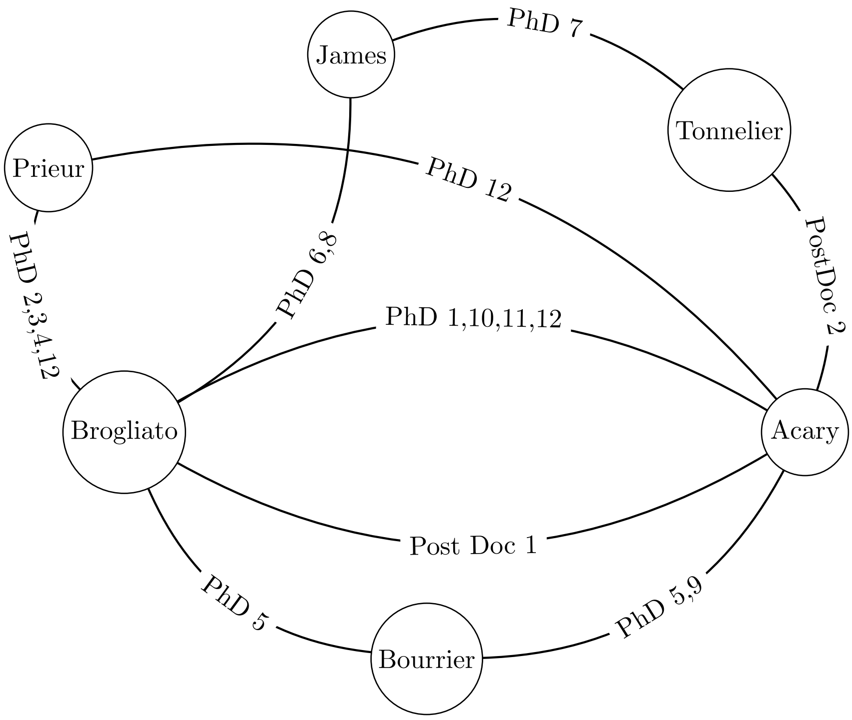


Figure 4: Graph of co-supervisions

4 Software developments and industrial transfer

4.1 Software platform: SICONOS

The aim of this development is to provide a common platform for the modeling, simulation, analysis and control of general nonsmooth dynamical systems. Besides usual quality attributes for scientific computing software, we want to provide a common framework for various scientific fields, to be able to rely on the existing developments (numerical algorithms, description and modeling software), to support exchanges and comparisons of methods, to disseminate the know-how to other fields of research and industry, and to take into account the diversity of users (end-users, algorithm developers, framework builders) in building user interfaces in Python and industry oriented applications.

In the framework of the FP5 European project Siconos (2002-2006), V. Acary was the leader of the Work Package 2 (WP2), dedicated to the numerical methods and the software design for nonsmooth dynamical systems. This gave rise to the platform Siconos in 2004 which was the main software development task in the Bipop team. We invested an important part of our activities developing new algorithms and maintaining the software architecture to answer to new challenges that come from applications. Siconos is now a mature software that can be used as a stand–alone software, as a module in Python or as a computational engine or library inside a package dedicated to a specific community. The software consists in around 370 000 lines of codes in C++, C, Fortran 77 and Python distributed under the Apache 2.0 license on github. The goal is to pursue this effort of development in TRIPOP.

Users community and assessment Siconos is used for research, education and by industrial partners with approximately 30 frequent users (200 registered users to the user-list "siconos-users"). In the Bipop Team, a large number of publications and also in the PhD thesis and post-doc fellows uses Siconos. For instance, in control's papers, there is no alternative to Siconos for the simulation of our system. Let us list the main use cases in other teams:

- Education : siconos is used for teaching simulation of nonsmooth systems at the University of Limoges.
- Research: B. Caillaud (Hycomes team) for simulation of hybrid an cyber-physical systems, S. Adly and H. Massias (XLIM, University of Limoges) for simulation of electrical circuits, P. Masareti and M. Fancello (Politecnico de Milano). siconos has been coupled to MBDyn (main open–source software for the simulation of multi-body–systems), C. Touzé (IMSIA, ENSTA ParisTech), C. Issanchou (Institut Jean Le Rond D'Alembert/UPMC). siconos is used for the simulation of strings with impacts in musical instruments to synthesize digital sounds , M. Ferris and O. Huber (University of Madison, Wisconsin). siconos is used for developing new algorithms for contact and friction based on PathVI approach, R. Kikuuwe (Kyushu University, Japan). siconos is used for control and simulation of nonsmooth robotic systems, Gazebo users. Through the work of S. Sinclair in the ADT Rope[6](Inria technological development action), siconos is coupled to gazebo[7] which is now a standard de facto for the simulation of robotic systems inside ROS (Robot Operating system[8])
- Industry: Schneider Electric uses siconos in production for virtual prototyping of circuit breakers, and in particular, for the robustness analysis of circuit breakers to manufacturing tolerances, Trasys Space for the simulation of the ExoMars Rover[9] of European Space Agency [6], Electricité de France uses siconos in the Saladyn project to simulate hydraulic dams made of concrete and rocky blocks, $\varphi-$ Ingeneria (Chile) uses siconos for the simulation of flows of granular material in ore processes.

Siconos increases also a lot our visibility. Without siconos, it would have been difficult to get our collaboration with ANSYS and to continue with Schneider Electric, and all–day use of the R&D engineers. In the same spirit, the book on switched electrical circuits and partly the book on numerical methods would not have been written. Up to now, this software is mainly used by experts through various interfaces. Our goal is to extend its use by interfacing Siconos with several open–source software codes that are standard or widespread in their own community (gazebo, modelica compiler, FreeCAD).

Description of the state of the art and placement siconos is the unique software that simulates general nonsmooth dynamical systems in a quite abstract form. This allows for applications from mechanical, electrical and control engineering to gene networks in biology. In computational mechanics, there are mainly three related research software codes.

- LMGC90 is a companion software of siconos for applications in mechanical engineering and multi–physic simulation. Now, we are seriously thinking how to merge our development, to increase the critical mass and to create a consortium to answer to industrial requests.
- MBsim[10]. This software, mainly developed at TUM (Technische Universität München) provides a subset of functionalities of Siconos concerning the simulation of multibody systems with contact and friction.
- chrono Engine[11] is an open–source tools for simulation of mechanical systems. Although the choice of solvers is quite restrictive, a massive investment by DARPA and NVidia through the SBEL lab[12] in developing GUI rendering tools and GPU parallel computation renders this project very attractive. Unfortunately, the European context renders difficult the same kind of development for siconos. We hope to bridge partly this gap thanks to a consortium with LMGC90.

[6] http://www.inrialpes.fr/bipop/people/acary/publications/Misc/ROPE_adt_2015-final.pdf
[7] http://gazebosim.org
[8] http://www.ros.org
[9] http://exploration.esa.int/mars/45084-exomars-rover/
[10] https://github.com/mbsim-env/mbsim
[11] http://chronoengine.info/chronoengine/
[12] http://sbel.wisc.edu

Priorities for the next 4 years The objectives for the next 4 years for the development of functionalities in Siconos are:

- Optimal control solvers for nonsmooth dynamics and state constraints.
- Space discretization and spectral methods of flexible mechanical systems in particular cable systems.
- Complete our integration in Gazebo.
- Numerical methods for rockfall trajectory (IRSTEA collaboration), geomaterials mainly granular materials (Material Point Method and Finite Element Method) for debris and mud flows, rock avalanches and the design of structural protections.
- Developments of various dedicated interfaces towards commercial software (ANSYS RDB, Dymola (Dassault systems)) and towards standard de facto for geotechnics.

Software's web site:

- General web site: http://siconos.gforge.inria.fr
- Developer's site: https://github.com/siconos/siconos
- Dashboard: http://cdash-bipop.inrialpes.fr
- Travis: https://travis-ci.org/siconos/siconos
- Youtube channel: https://www.youtube.com/channel/UCgv2siTCJeSdWFPTDk71Iyw

4.2 Industrial transfer

Prototyping of multibody systems (Schneider Electric, European Space Agency (ESA), ANSYS)

Participants: V. Acary, B. Brogliato, F. Pérignon, M. Brémond, G. James

With Schneider Electric, we want to apply the new methods for vibrations, modes and quasi-periodic solutions in nonsmooth flexible mechanical systems to the study of the robustness of circuit breakers to vibratory environments and seismic excitations, in order to answer to recent more stringent international norms.

Cableways transportation (STRMTG, POMA)

Participants: V. Acary, B. Brogliato, M. Brémond, G. James, C. Prieur

We have recently contacted both STRMTG and the POMA company about modelling, simulation and control of cable-transport systems. In such systems, the question of the coupling between the nonlinear dynamics of cables and their supports with unilateral contact and friction appears now to be determinant in order to increase the performances of the cableway systems, especially for urban transportation systems.

Natural hazards (IMSRN, INDURA) and Mining industries (Inria Chile, Codelco, Timining)

Participants: F. Bourrier, V. Acary, B. Brogliato, G. James, S. Sinclair, S. Candela

Through a starting collaboration with F. Bourrier (IRSTEA Grenoble), we will use our software siconos to assess the natural risk related to rockfalls and the rock slope stability. These questions are also directly related to the applications to mining industries that we want to continue with Inria Chile. More generally, the question of granular flows, rock fracture and rock pre-conditioning in mining industries will provide us with very interesting applications of our simulation techniques.

Cyber-physical modeling and Simulation (Dassault Systems)

Participants: V. Acary, B. Brogliato, M. Brémond, A. Tonnelier

In the framework of the Inria IPL Modeliscale and in a FUI project in construction in the Systematic cluster (Paris Region Digital Ecosystem), these research developments will be used and implemented by Dassault Systems in the dymola software, and used by EdF for modeling smart grids and piping systems. We have started discussions with them to understand exactly what are their priorities. We want together to extend the modelica toolboxes with nonsmooth components in mechanics, electronics, hydraulics and control.

5 Institutional context and positioning

5.1 Positioning inside Inria

We are the only team at Inria dealing all these aspects of nonsmooth systems with strong skills on complementarity problems and variational inequalities, their analysis, control and numerical simulation.

In the framework of the Inria Project Lab (IPL) and the FUI Modeliscale, we strongly collaborate with HYCOMES (Inria Rennes Atlantique, B. Cailllaud), PARKAS (Inria Paris, M. Pouzet) on the analysis, simulation of cyberphysical systems. With the NON-A (POST) team (A. Polyakov, D. Efimov), we jointly work on the implicit discretization for sliding-mode control and observation. In 2018, an ANR project DIGITSLID (coordinator B. Brogliato) has been submitted to continue this effort.

Other Inria teams with which our scientific activities have non void intersection are:

- COMMANDS (Inria Paris-Saclay): optimal control with constraints.
- IBIS (Inria Grenoble), BIOCORE (Inria Sophia Antipolis): control and simulation of piecewise linear systems.
- HEPHAISTOS (Inria Sophia Antipolis): control of cable-driven systems.
- DEFROST (Inria Lille): control of deformable mechanical systems.
- NON-A (Inria Lille):
- POEMS (Inria Paris): wave propagation and nonlinear wave analysis.
- SPHINX (Inria Nancy): control and stabilization of partial differential equations.
- MATHNEURO (Inria Sophia Antipolis)

5.2 Local and national contexts

Again, we do not know other multi-disciplinary teams in France that work on all the aspects of nonsmooth dynamical systems (modeling, analysis, simulation and control) in a general setting.

In this section, we list the French laboratories that work on nonsmooth dynamical systems, however focusing on one aspect only. We classify them by scientific domains, and we specify laboratories with which we have collaborations, and the other that could be considered as future collaborators.

Mechanical engineering and computational mechanics We actively work with the following laboratories, which represent the main actors in the domain of mechanical systems with contact and friction:

- LMGC (P. Alart, F. Dubois): simulation and analysis of granular material, software development.
- LTDS-ENTPE (C. Lamarque): nonlinear dynamics of cables with contact and friction within the co-supervision of the PhD thesis of C. Bertrand.

Another collaboration on musical instruments with contact exists with others French labs that are not expert on nonsmooth mechanics, but interested in this aspect for applications:

- Institut Jean Le Rond dAlembert, Equipe LAM, Sorbonne Université. J.L. Le Carrou
- IMSIA, ENSTA ParisTech-CNRS-EDF-CEA, Universite Paris Saclay, Palaiseau. C. Touzé.

Possible collaborations could be considered with other labs that are not expert on nonsmooth mechanics, on the following aspects: LAUM (Laboratoire d'Acoustique de l'Université du Maine) (granular crystals), LMA (Laboratoire de Mécanique et d'Acoustique) (nonlinear modes in mechanics, B. Cochelin) and LIRMM (cable-driven systems, M. Gouttefarde),

Hybrid and Cyber-Physical systems In the framework of the Inria Project Lab (IPL) and the FUI Modeliscale, we collaborate with groups outside INRIA:

- L2S (A. Girard): cyberphysical systems analysis and control.
- VERIMAG (G. Frehse) cyberphysical systems verification

Mathematical analysis and control

- LAAS (A. Tanwani): analysis, control, state observation of set-valued dynamical systems, Lur'e set-valued systems.

- LS2N, Ecole Centrale de Nantes (F. Plestan, Y. Aoustin, M. Ghanes): discrete time sliding–mode control.
- Laboratoire PIMENT, université de la Réunion (D. Goeleven): stability and analysis of nonsmooth dynamical systems.
- Laboratoire XLIM, universit de Limoges (S. Adly, P. Armand): mathematical analysis of nonsmooth systems and numerical optimization for SOCCP.

5.3 International context

A non-exhaustive list of people whose scientific interests partly match with ours (those with whom we have or had collaborations are marked with (**)):

- (**) University of Stuttgart, Mechanical Engineering (R.I. Leine): nonsmooth mechanical systems analysis and control, impact mechanics.
- University of Groningen (K. Camlibel): Lur'e set-valued systems, dissipativity and complementarity.
- Technology University of Eindhoven (N. van de Wouw): control of nonsmooth systems.
- University of Bristol (A. Champneys, N. Hogan): analysis of Painlevé paradoxes.
- (**) University of Naples (A. Frasca, L. Iannelli): complementarity systems analysis and computation.
- (**) Kyushu University (R. Kikuuwe) and Tokyo Institute of Technology (Y. Kanno): discrete-time sliding-mode control and observers, optimization of structures.
- Rensselaer Polytechnic Institute, Mathematics of robotics (J. Trinkle): complementarity in robotics.
- (**) McGill University, Mechanical Engineering (M. Legrand, J. Kovecses): nonlinear modes in nonsmooth mechanical systems, numerical solvers for contact mechanics, impact mechanics. J. Kocveses was invited professor for 2 months in 2017 in the BIPOP team, and M. Legrand will be hosted 3 months in 2018 in the TRIPOP team.
- (**) Technion, Mechanical Engineering (Y. Starosvetsky): nonlinear waves in discrete mechanical systems.
- (**) Université de Liège, Aerospace and Mechanical Engineering (O. Brüls) and Centro de Investigación de Métodos Computacionales (CIMEC), Sante FE, Argentina (A. Cardona): numerical analysis for nonsmooth systems and DAEs.
- (**) Universidad de Santiago de Chile, Mathematics Department, Universidad Técnica Federico Santa María (Juan Peypouquet, Eduardo Cerpa): algorithms for constrained variational inequalities, optimization problems, control and stabilization of nonlinear partial differential equations.
- (**) Cinvestav Mexico, Departamento de Control Automàtico (F. Castanos): passivity-based control of constrained mechancal systems, discrete-time sliding-mode control, robust control and maximal monotone differential inclusions.
- Universita di Parma (A. Tasora): mechanics with impacts and friction, software development (CHRONO).
- University of Wisconsin at Madison (M. Ferris): optimization algorithms for MPEC.
- (**) Peking University PKU (C. Liu): impact mechanics.

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