Numerical methods for Nonsmooth Dynamical Systems

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Contents

Objectives

- ▶ Be acquainted with the standard notation and definitions of nonsmooth dynamical systems.
- ▶ Be more familiar with robust numerical algorithms for the time–integration of nonsmooth dynamical systems: time–stepping methods and event-driven methods.
- ▶ Be able to formalize discretized system in standard forms prone to using optimization techniques.
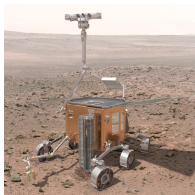- ▶ Know what are the main families of solvers for discrete problems.
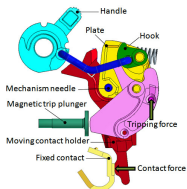
Nonsmooth dynamical systems

nonsmooth = lack of continuity/differentiability
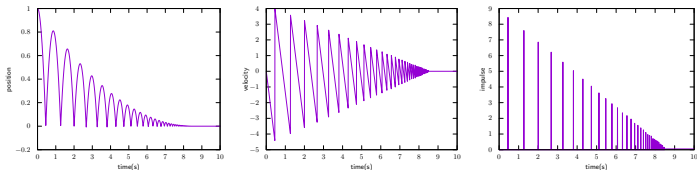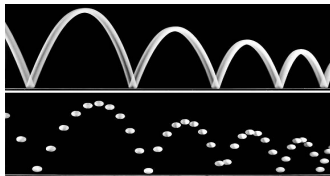
- ▶ nonsmooth solutions in time (jumps, kinks, distributions, measures)
- ▶ nonsmooth modeling and constitutive laws (set–valued mapping, inequality constraints, complementarity, impact laws)

Application fields.

- **Computational mechanics**. Plasticity. Unilateral contact, Coulomb friction and impacts : multi-body systems, robotic systems, frictional contact oscillators, granular materials.
- **Electronics**. Switched electrical circuits (digital/analog converters and power electronics, diodes, transistors, switchs).
- **Computer science**. Hybrid and Cyber–physical systems
- **Bio-mathematics**. Gene regulatory networks
- **Transportation science**. Fluid transportation networks with queues.
- **Economy and Finance**. Oligopolistic market equilibrium

Nonsmooth approach is crucial for a correct modeling and a efficient simulation

Sources of nonsmoothness

▶ Two largely different time-scales of evolution:

  1. a slow smooth dynamics (free flight of the bouncing ball)
  2. a very fast dynamics (events, transitions, impacts) that can be modeled as a punctual event.
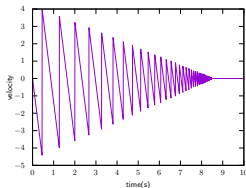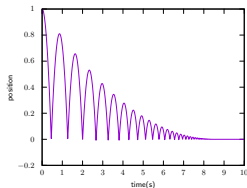
Nonsmooth dynamical systems

Difficulty
Standard tools of numerical analysis and simulation (in finite dimension) are no longer suitable due to the lack of regularity.

Specific tools
Differential measure theory. Convex, nonsmooth and variational Analysis (Clarke, Wets & Rockafellar). Complementarity theory. Maximal monotone operators.

Examples of nonsmooth dynamical systems
▶ Piecewise smooth systems
▶ Complementarity systems and differential variational inequality.
▶ Specific differential inclusions (Filippov, Moreau sweeping process, Normal cone inclusion).