Time integration of nonsmooth mechanical systems with unilateral contact. Conservation and stability of position and velocity constraints in discrete time.

Vincent Acary, O. Bonnefon
INRIA Rhône–Alpes, Grenoble.

ENOC 2011, Roma, Italy. 25–29 July 2011.

vincent.acary@inrialpes.fr

Introduction & Motivations

Outline

A naive projection scheme

A projection/activation scheme

Conclusions & Perspectives

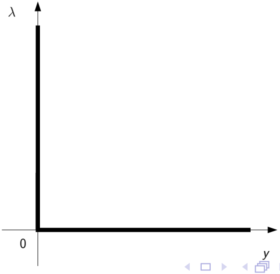
NonSmooth Multibody Systems

Scleronomous holonomic perfect unilateral constraints

$$
\begin{cases}
M(q(t))\dot{v} = F(t, q(t), v(t)) + G(q(t))\,\lambda(t), \text{ a.e} \\
\dot{q}(t) = v(t), \\
g(t) = g(q(t)), \quad \dot{g}(t) = G^T(q(t))v(t), \\
0 \leqslant g(t) \perp \lambda(t) \geqslant 0, \\
\dot{g}^+(t) = -e\dot{g}^-(t),
\end{cases}
\tag{1}
$$

where $G(q) = \nabla g(q)$ and e is the coefficient of restitution.

Unilateral constraints (unilateral contact, Signorini condition)

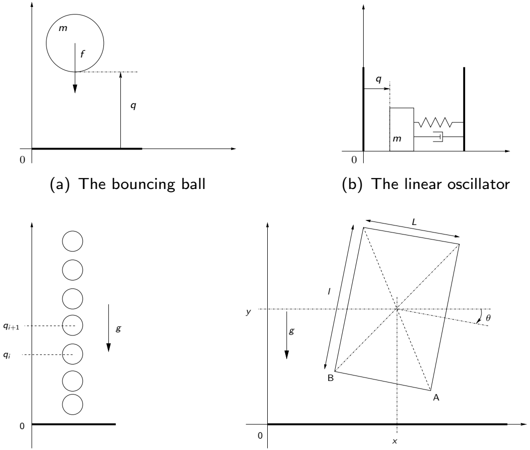


Vincent Acary, O. Bonnefon
INRIA Rhône–Alpes, Grenoble.

Introduction & Motivations
Problem setting
Objectives & means
Outline
A naive projection scheme
A projection/activation scheme
Conclusions & Perspectives

NonSmooth Multibody Systems (NSMBS)

Academic examples



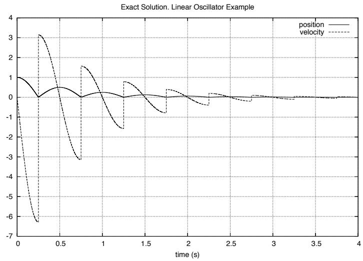
(a) The bouncing ball (b) The linear oscillator

(c) The chain of balls (d) The rocking block

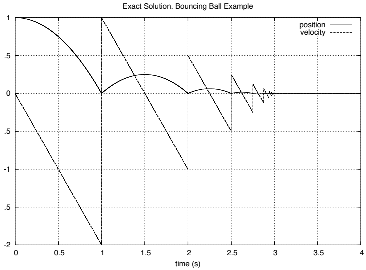
Figure: Simple archetypal test examples

Introduction & Motivations
Problem setting
Objectives & means
Outline
A naive projection scheme
A projection/activation scheme
Conclusions & Perspectives

NonSmooth Multibody Systems (NSMBS)



(a) Linear Oscillator [?]

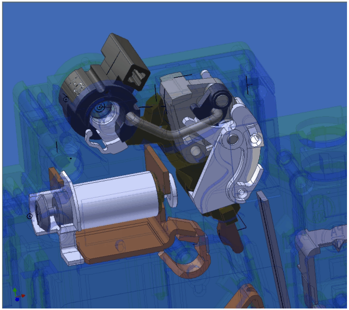


(b) Bouncing ball example

Figure: Analytical solutions.

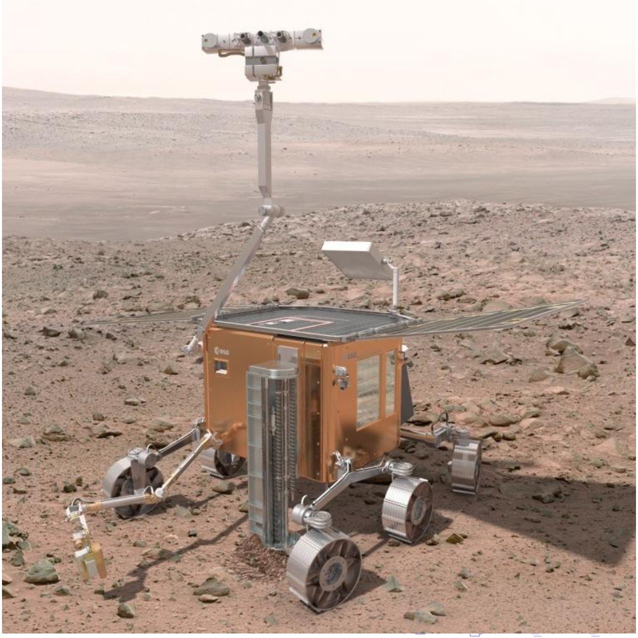
Mechanical systems with contact, impact and friction

Simulation of Circuit breakers (INRIA/Schneider Electric)



Mechanical systems with contact, impact and friction

Simulation of the ExoMars Rover (INRIA/Trasys Space/ESA)



Vincent Acary, O. Bonnefon
INRIA Rhône–Alpes, Grenoble.

Introduction & Motivations
Problem setting
Objectives & means
Outline
A naive projection scheme
A projection/activation scheme
Conclusions & Perspectives

State–of–the–art

Numerical time–integration methods for Nonsmooth Multibody systems (NSMBS):

Nonsmooth event capturing methods (Time–stepping methods)

- $\oplus$ robust, stable and proof of convergence
- $\oplus$ low kinematic level for the constraints
- $\oplus$ able to deal with finite accumulation
- $\ominus$ very low order of accuracy even in free flight motions

Two main implementations

- Moreau–Jean time–stepping scheme
- Schatzman–Paoli time–stepping scheme

Moreau's Time stepping scheme [??]

Principle

$$\left\{\begin{array}{ll} M(q_{k+\theta})(v_{k+1}-v_k) - hF_{k+\theta} = p_{k+1} = G(q_{k+\theta})P_{k+1}, & (2a) \\ q_{k+1} = q_k + hv_{k+\theta}, & (2b) \\ U_{k+1} = G^T(q_{k+\theta})\,v_{k+1} & (2c) \\ \left.\begin{array}{ll} 0 \leqslant U^{\alpha}_{k+1} + eU^{\alpha}_{k} \perp P^{\alpha}_{k+1} \geqslant 0 & \text{if} \quad \bar{g}^{\alpha}_{k,\gamma} \leqslant 0 \\ P^{\alpha}_{k+1} = 0 & \text{otherwise} \end{array}\right. . & (2d) \end{array}\right.$$

with

- $\theta \in [0,1]$
- $x_{k+\theta} = (1-\theta)x_{k+1} + \theta x_k$
- $F_{k+\theta} = F(t_{k\theta}, q_{k+\theta}, v_{k+\theta})$
- $\bar{g}_{k,\gamma} = g_k + \gamma h U_k, , \gamma \geqslant 0$ is a prediction of the constraints.

Schatzman's Time stepping scheme [?]

Principle

$$\begin{cases} M(q_k+1)(q_{k+1}-2q_k+q_{k-1}) - h^2 F_{k+\theta} = p_{k+1}, & (3a) \\ v_{k+1} = \dfrac{q_{k+1}-q_{k-1}}{2h}, & (3b) \\ -p_{k+1} \in N_K\left(\dfrac{q_{k+1}+eq_{k-1}}{1+e}\right), & (3c) \end{cases}$$

where N_K defined the normal cone to K.
For $K = \{q \in \mathbb{R}^n, y = g(q) \geqslant 0\}$

$$0 \leqslant g\left(\frac{q_{k+1}+eq_{k-1}}{1+e}\right) \perp \nabla g\left(\frac{q_{k+1}+eq_{k-1}}{1+e}\right) P_{k+1} \geqslant 0 \qquad (4)$$

Comparison

Shared mathematical properties

- Convergence results for one constraints
- Convergence results for multiple constraints problems with acute kinetic angles
- No theoretical proof of order

Mechanical properties

- Position vs. velocity constraints
- Respect of the impact law in one step (Moreau) vs. Two-steps(Schatzman)
- Linearized constraints rather than nonlinear.

But

These schemes do not satisfy both the position and velocity constraints in discrete time.

Introduction & Motivations
Problem setting
Objectives & means
Outline
A naive projection scheme
A projection/activation scheme
Conclusions & Perspectives

Objectives & means

Objectives

Design nonsmooth event capturing (time–stepping) methods with

- Same properties as standard methods (robustness, accumulation, . . .)
- Controlled drift of the constraints also for the perfect bilateral constraints
- Study of multi–body systems with clearances. (Influence of clearance on stability, avoiding violations of the same of order of clearances)
- Consistency with CAD tool and detection collision engines (which suffer even from slight violations)

Means

- Projection algorithms onto the manifold defined by the constraints

Vincent Acary, O. Bonnefon
INRIA Rhône–Alpes, Grenoble.

Introduction & Motivations
Problem setting
Objectives & means
Outline
A naive projection scheme
A projection/activation scheme
Conclusions & Perspectives

Illustrations

Pendulum in a ring

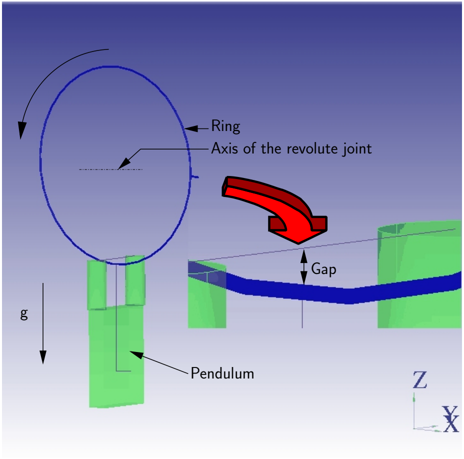


Figure: Pendulum in a ring

Vincent Acary, O. Bonnefon
INRIA Rhône–Alpes, Grenoble.

Introduction & Motivations
Problem setting
Objectives & means
Outline
A naive projection scheme
A projection/activation scheme
Conclusions & Perspectives