Modification of Moreau-Jean's Scheme for Energy Conservation in Inelastic Impact Dynamics

Carlos Yoong [1] Vincent Acary [2] Mathias Legrand [1]

[1] *Structural Dynamics and Vibration Laboratory*
McGill University

[2] Team Bipop
Inria Rhone-Alpes

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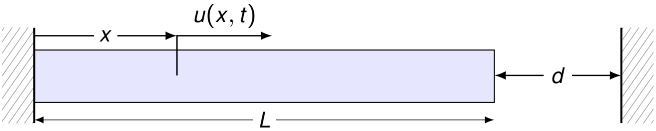
Motivation

General Motivations

- Find periodic solutions for continuous systems with sticking contact phases
- Perform nonsmooth modal analysis (frequencies of vibration and mode shapes)

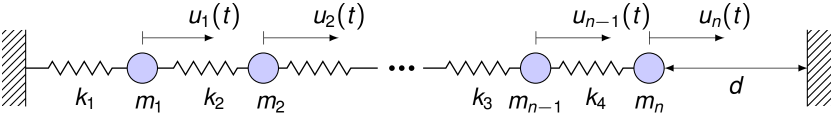


Issues with standard space discretization (FEM or FD)

- Finite dimensional systems require impact laws.
- Sticking phases implies inelastic impacts ➜ dissipative system.
- Develop a scheme that conserves energy for inelastic impact dynamics

Vibro-impact system

Mechanical model



Dynamics of the system

- Equations of motion

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{r} \tag{1a}$$

$$\mathbf{u}(0) = \mathbf{u}_0, \quad \dot{\mathbf{u}}(0) = \dot{\mathbf{u}}_0 \tag{1b}$$

$$u_n(t) \leq d, \quad R(t) \leq 0, \quad (u_n(t) - d)R(t) = 0, \quad \forall t \tag{1c}$$

- Newton's impact law

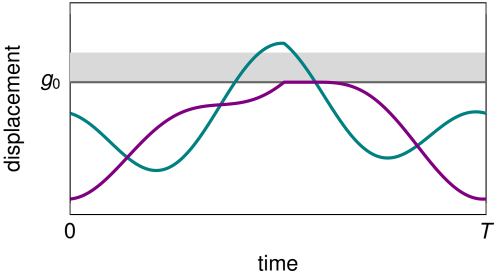
$$\dot{u}_n(t^+) = -e\dot{u}_n(t^-) \quad \text{if} \quad u_n(t) = d \quad \text{and} \quad \dot{u}_n(t^-) \geq 0. \tag{2}$$

Vibro-impact system

Purely inelastic impact law

- Coefficient of restitution: $e = 0$
- Dissipation of energy
- System undergoes "sticking" contact phases
- Periodic solutions $\Longleftrightarrow$ conservation of energy

Illustration of sticking contact phases for 2 DOF system



Moreau-Jean's Scheme

Numerical time integration

- Within interval $(t^i, t^{i+1}]$ of length h and using $\theta \in [0, 1]$

$$\mathbf{M}\left(\dot{\mathbf{u}}^{i+1} - \dot{\mathbf{u}}^{i}\right) + h\mathbf{K}\mathbf{u}^{i+\theta} = \mathbf{p}^{i+1}, \tag{3a}$$

$$\mathbf{u}^{i+1} = \mathbf{u}^{i} + h\dot{\mathbf{u}}^{i+\theta}, \tag{3b}$$

$$\text{if } \; u_n^i \geq d, \quad 0 \leq \dot{u}_n^{i+1} + e\dot{u}_n^i \perp P^{i+1} \leq 0 \tag{3c}$$

Numerical energy dissipation

- Discrete time dissipation equality [Acary, ZAMM, 2015]

$$\Delta\mathcal{E} = (\frac{1}{2} - \theta)\left[||\dot{\mathbf{u}}^{i+1} - \dot{\mathbf{u}}^{i}||_M^2 + ||\mathbf{u}^{i+1} - \mathbf{u}^{i}||_K^2\right] + \dot{u}_n^{i+1/2} P^{i+1} \tag{4}$$

- Most conservative case when $\theta = 1/2$, the total energy balance reads:

$$\Delta\mathcal{E} = \mathcal{E}^{i+1} - \mathcal{E}^{i} = \dot{u}_n^{i+1/2} P^{i+1} \tag{5}$$

Correction for energy conservation

Proposition

- Contact closes $\Rightarrow$ Add scalar correction β^{i+1} to velocity of non-contacting masses.
- β^{i+1} is not know a priori and is calculated for each time contact closes.
- The correction is calculated from:

$$\frac{1}{2}\left(\mathbf{I}\beta^{i+1}\right)^{\top}\mathbf{M}\left(\mathbf{I}\beta^{i+1}\right)+\left(\mathbf{I}\beta^{i+1}\right)^{\top}\mathbf{M}\dot{\mathbf{u}}^{i+1}+\frac{h^2}{8}\left(\mathbf{I}\beta^{i+1}\right)^{\top}\mathbf{K}\left(\mathbf{I}\beta^{i+1}\right)+\frac{h}{2}\left(\mathbf{I}\beta^{i+1}\right)^{\top}\mathbf{K}\mathbf{u}^{i+1}+\dot{u}_2^{i+1/2}P^{i+1}=0. \tag{6}$$

- For min $\left|\beta^{i+1}\right|$, the system does not present infinite sticking phases nor a behavior involving purely elastic impacts.

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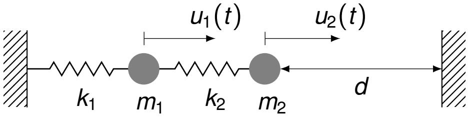
Drawbacks

- No rigorous procedure to chose the correct β^{i+1}
- Numerical dispersion errors are not corrected

1. Motivation
2. Vibro-impact system
   - Mechanical model
   - Purely inelastic impact law
3. Moreau-Jean's scheme with energy correction
4. Results
   - Vibro-impact responses with sticking contact phases
   - Families of periodic solutions
5. Conclusions and Future Work

Vibro-impact system

System of interest



Mechanical properties

- Mechanical properties: $k_1 = k_2 = 1$ and $m_1 = m_2 = 1$.
- Natural frequencies: $\omega_1 = 0.6180$ and $\omega_2 = 1.6180$
- Natural periods: $T_1 = 10.1670$ and $T_2 = 3.8833$
- Initial conditions: $\mathbf{u}_0 = (0 \;\; 0)^\top$ and $\dot{\mathbf{u}}_0 = (10 \;\; 10)^\top$
- Initial gap: $d = 0.1$

Simulation parameters

- Time step: $h = 1/500$

Responses with sticking contact phases

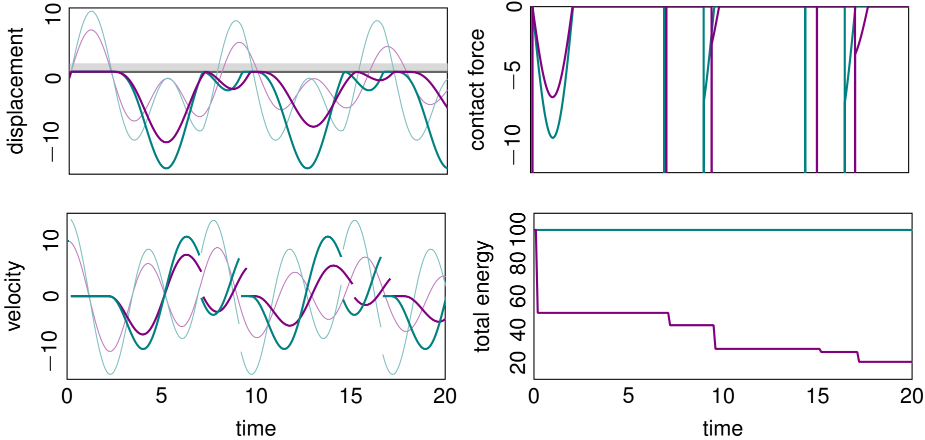
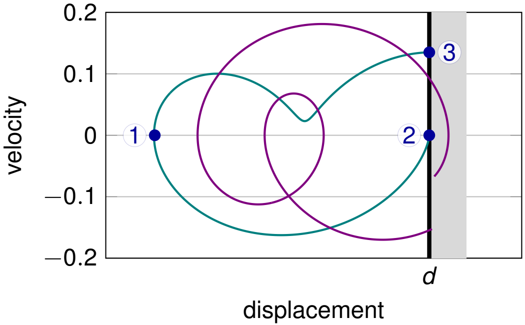


Figure: Vibro-impact dynamics with $e = 0$. Green color: modified scheme. Purple color: standard scheme. For displacement and velocity plots: light colors depict mass 1 and dark colors depict mass 2.

Computation of periodic solutions

Shooting method with continuation

- Newton's method with finite difference Jacobian matrix
- Sensitive to numerical errors $\rightarrow$ Difficult to converge
- Continuation along energy by taking point 1 as reference
- Points 2 and 3 never converge due to penetration



Families of periodic solutions

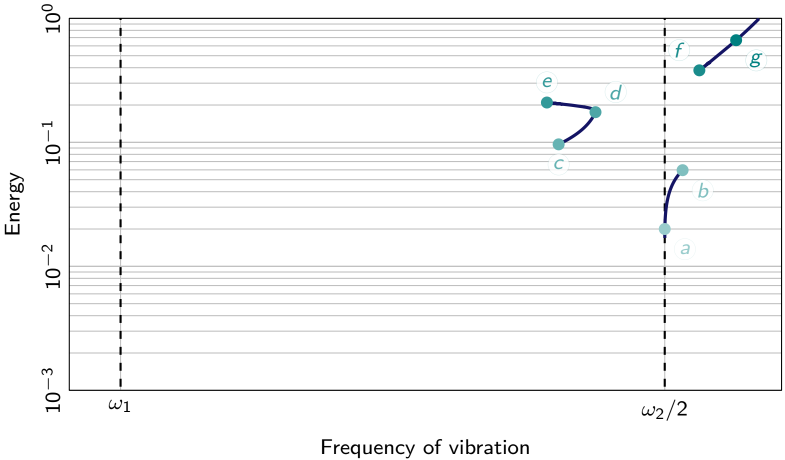


Figure: Frequency-Energy plot for families of periodic solutions with sticking contact phases.

Families of periodic solutions

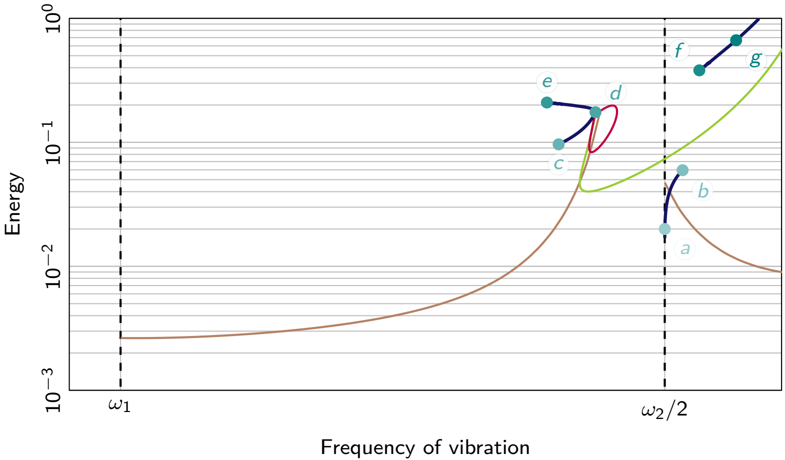
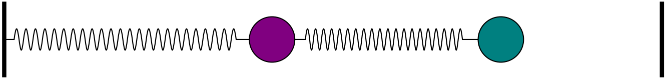


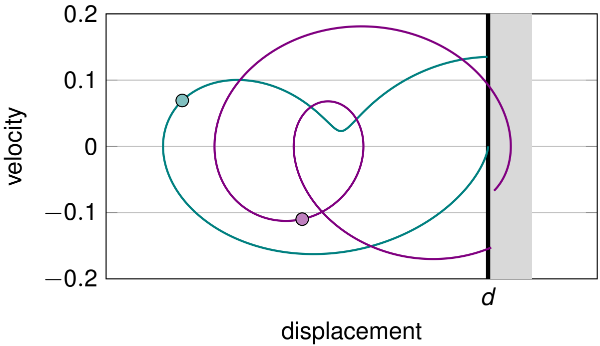
Figure: Blue: periodic solutions with sticking contact phases. Orange: periodic solutions with purely elastic impact law ($e = 1$) for one impact per period. Light green: two impacts per period ($e = 1$). Red: three impacts per period ($e = 1$).

Periodic Solution a

Animation of the system

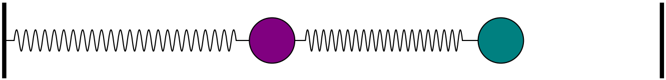


Phase diagram

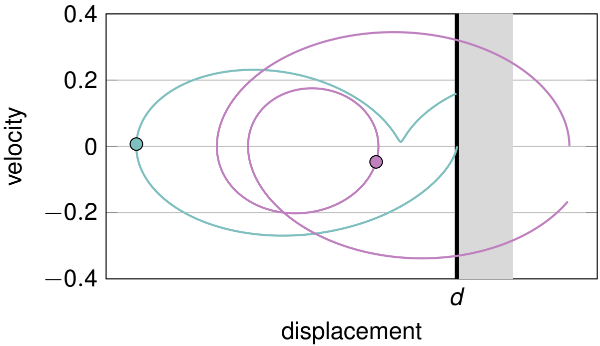


Periodic Solution b

Animation of the system

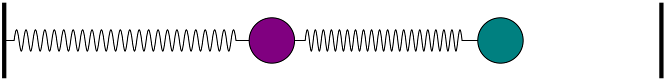


Phase diagram

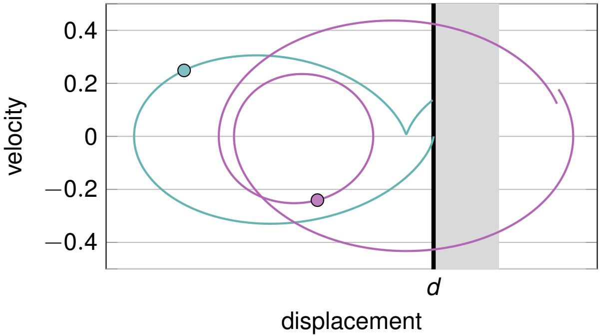


Periodic Solution c

Animation of the system

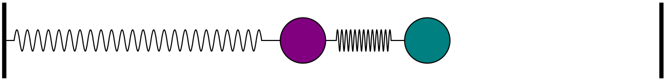


Phase diagram



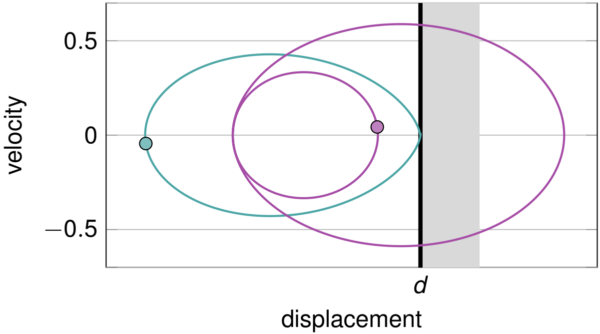
Periodic Solution d

Animation of the system



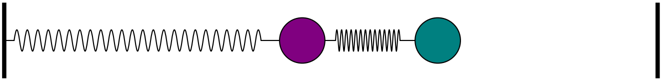
Phase diagram

- Solution also found in an analytical study by Le Thi et al.

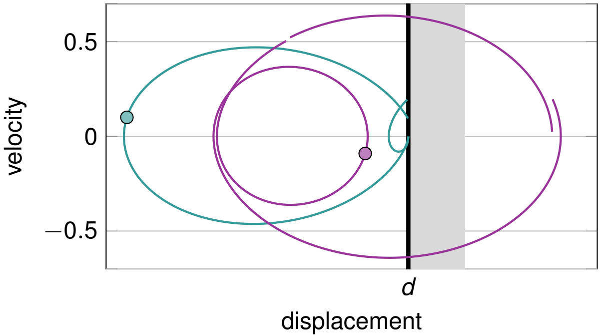


Periodic Solution e

Animation of the system

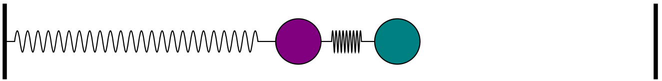


Phase diagram

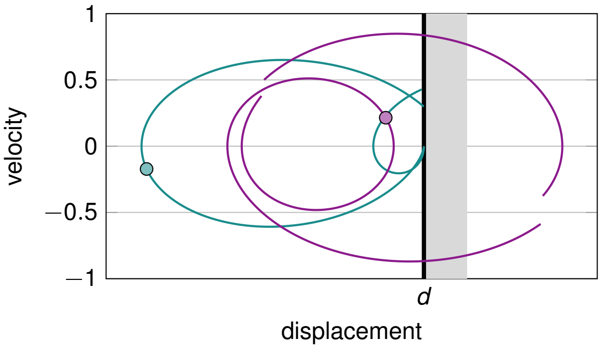


Periodic Solution f

Animation of the system

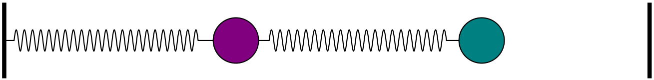


Phase diagram

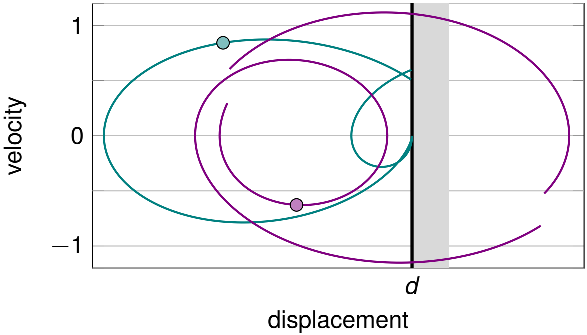


Periodic Solution g

Animation of the system



Phase diagram



Conclusions and Future Work

Conclusions

- Inclusion of the correction yields an energy conserving framework
- Results converge to periodic solutions with sticking contact phases
- More than 2 DOF are highly affected by numerical dispersion

Future Work

- Include a correction in displacement and/or velocity
- Eliminate numerical dispersion
- Find periodic solutions in multidimensional continuous systems