Biol. Cybern. 92, 303–315 (2005)
DOI 10.1007/s00422-005-0553-3

Biological
Cybernetics

A model of contextual effect on reproduced extents in recall tasks: the issue of the imputed motion hypothesis

Jean-Christophe Sarrazin, Arnaud Tonnelier, Frederic Alexandre

U.M.R. 7503: LORIA - INRIA / Lorraine, Campus Scientifique - BP 239, 54506 Vandoeuvre-lès-Nancy Cedex, France

Received: 2 June 2004 / Accepted: 2 February 2005 / Published online: 18 April 2005

Abstract. In this article the fundamental question of space and time dependencies in the reproduction of spatial or temporal extents is studied. The functional dependence of spatial responses on the temporal context and the corresponding dependence of temporal responses on spatial context are reported as the tau and kappa effects, respectively. A common explanation suggested that the participant imputes motion to discontinuous displays. Using a mathematical model we explore the imputed velocity hypothesis and provide a globally fit model that addresses the question of sequences modelling. Our model accounts for observed data in the tau experiment. The accuracy of the model is improved introducing a new hypothesis based on small velocity variations. On the other hand, results show that the imputed velocity hypothesis fails to reproduce the kappa effect. This result definitively shows that both effects are not symmetric.

1 Introduction

How do space and time relate in a recall task of spatial or temporal extents? And how could a mathematical model account for the observed data? The present article addresses these fundamental questions that are of long-standing interest to experimental psychology. Indeed, the relation between spatial and temporal dimensions of stimuli was addressed in studies of perception in the first half of the last century (Benussi 1913; Helson 1930; Helson and King 1931), and, more recently, in the domain of motor control (Lee 2000). Links between spatial and temporal dynamics of action have been documented in the framework of precision aiming tasks (Fitts 1954; Meyer et al. 1988; Mottet and Bootsma 1999) and bi-manual coordination tasks (Kelso 1981; Temprado et al. 2001; Zanone and Kelso 1992).

Correspondence to: J.-C. Sarrazin
(e-mail: sarrazin@univ-tln.fr,
Tel.: +33-3-83592056 Fax: +33-3-83278319)

1.1 The issue of space–time organization

It is customary to acknowledge that processing of visual information involves both spatial and temporal dynamics. However, studies of the memorization of visual configurations have generally focused on the spatial characteristics of reproduced configurations (for a review, see Finke and Shepard 1986), often to the neglect of their temporal dimension. Indeed, researchers often consider space and time as two distinct conceptual and methodological fields of study. From this point of view memorizing a pattern and memorizing a sequence are two different issues. However Gibson (1966) showed that spatial and temporal distributions are related. Hence, the perception of space cannot be fully understood unless we tackle it as a problem of space-time. Spatial and temporal aspects are interdependent, are not separable, and together determine the event (Schill and Zetzsche 1995).

The issue of the relation between the spatial and temporal dimensions of stimuli was already raised in the first half of the twentieth century in studies on perception. In what was probably the first experiment of this type, Benussi (1913) presented participants with three successive flashes of light defining two spatial and two temporal inter-stimulus intervals (spatial ISIs for spatial inter-stimulus interval, te mporal ISIt for temporal inter-stimulus interval). Distance judgements were found to vary as a function of the duration of the temporal ISIt. Since then, many studies – involving the visual (Bill and Teft 1969), auditory (Cohen et al. 1953), or kinesthetic (Helson and King 1931; Lechelt and Bochert 1977) modality – have shown that when two constant spatial ISIs are associated with variable temporal ISIt, distance is overestimated or underestimated in accordance with the temporal ISIt, an effect Helson (1930) coined the "tau effect".

It has long been known that the perception of brief temporal intervals is influenced by the context in which they are presented. Using the methodology of Benussi (1913), with three light flashes indicating two distances and two durations, Abe (1935) demonstrated that judgements of duration varied according to the distances defined by the spatial ISIs. When temporal ISIt are constant and spatial

ISIs are variable, the temporal judgements vary under the effect of the spatial ISIs, a phenomenon termed the "kappa effect" by Cohen et al. (1953).

In other words, the space–time dependencies are clearly illustrated by the tau and kappa effects that are deformations that arise when subjects judge the distance (tau) or the duration (kappa). More precisely, the tau effect occurs when constant distances between stimuli delivered at variable time intervals are perceived as variable distances. The kappa effect is the corresponding dependence of the temporal judgment on the spatial context.

1.2 The issue of the imputed velocity hypothesis

Price-Williams (1954) suggested, in explanation of the kappa effect, that the participant imputes motion to the static display, and Cohen et al. (1955) argued that separate spatial or temporal discriminations are based on the primacy of discrimination of movement. Clarifying the notion of an imputed motion, Cohen et al. (1955) suggested that the reference to discrimination of movement is perhaps misleading. The imputed velocity hypothesis does not suggest that the participant in a tau or kappa experiment perceives the display as in motion or that apparent motion is experienced. In fact, Mashour (1964) explicitly pointed out that experience of apparent motion results in participants showing the reverse of the tau effect (i.e., when the spatial dimension to be reproduced is variable and the temporal dimension to be ignored is constant). In this context, Jones and Huang (1982) suggested that "imputed motion means simply that a participant who is required to make judgments about ambiguous spatial or temporal interval makes a decision based on the familiar functional relations between distance (s), time (t), and average velocity (v)" (p.134). As Collyer (1977) noted, the tau and kappa effects are consistent with the assumption that the participant, given two successive spatial and temporal intervals, equalizes the ratio t_1/t_2 and s_1/s_2 . It follows obviously that $s_1/t_1 = s_2/t_2$ or that $v_1 = v_2$. In this context, Anderson (1974) and Collyer (1977) suggested that both effects reflect a linear combination of spatial and temporal variables. In the case of the tau effect, the participant's observed response is taken to be the weighted average of the given distance and the expected distance that would be required to traverse the given distance at constant velocity. The argument can be generalized easily to the kappa effect. Following this argument, Jones and Huang (1982) developed an algebraic model to account for the tau and kappa effects observed in their psychological data. Their model is well-adapted for the description of the effects in small sequences (two space-time intervals) but it is not clear how it could be generalized to larger sequences.

The aim of this study is to address the following questions: (1) Does the imputed motion hypothesis explain the observed tau and kappa effects in large sequences reproduction? (2) How does a mathematical model account for a description of these effects? (3) Are the tau and kappa effects symmetric? To answer these questions we rely on a part of the Sarrazin et al. (2004) experimental study. We develop a mathematical framework of the tau and kappa effects in recall tasks of large space-time patterns (i.e., characterized by more than two intervals). This model offers a critical view of the imputed velocity hypothesis. The paper is organized as follows. In Sect. 2, we present the experimental conditions studied by Sarrazin et al. (2004) and restrict our attention to the experiments where the tau or kappa effects occurred. In a third section, we give a mathematical formulation of the imputed velocity hypothesis. We focus on the well-known constant velocity hypothesis and suggest an improvement through a so-called minimum acceleration hypothesis. Implications of these models are discussed. In Sect. 4, the model's predictions are presented and compared with the experimental responses. Finally, we provide a discussion of our results.

2 Experiment: the case of complex space–time patterns in recall tasks

Transposing the interactive effects of space and time to the study of recalling a complex pattern (e.g. including more than three stimuli sequentially presented), Sarrazin et al. (2004) set out to compare reproduction performance for configurations in which the spatial and temporal ISI were not proportional so that the configuration was presented at variable speeds (certain distances were covered more slowly or more rapidly). After a learning phase, the participants were asked to situate the configuration elements as accurately as possible. The question raised concerned the effect of temporal context on the characteristics of spatial encoding (i.e., the tau effect). When the task of the participant was no longer to remember the elements' location but to reproduce the temporal pattern (i.e., the rhythm), the question raised concerned the effect of spatial incongruence on temporal encoding (i.e., the kappa effect). The goal of these experiments was to determine whether the tau and kappa effects were present in recall tasks involving reproduction of the spatial dimension or the temporal dimension of complex space-time patterns. Let us now detail the method and present the experimental results.

2.1 Method

2.1.1 Participants Twenty four adult volunteers (12 women and 12 men) participated in this experiment. Their mean age was 24 years (range = 20–38 years). Participants were divided into two groups. The first group was asked to reproduce the spatial characteristics of the target configuration and the second group was asked to reproduce the temporal characteristics.

2.1.2 Material The stimuli consisted of a set of eight white dots (0.4 cm in diameter, subtending 0.13 angular degrees) presented against a black background. Dots appeared, one at the time, on a 17-inch screen located 1.70 m from the participant so that the configuration appeared in perifoveal vision (i.e., without necessitating head movement). Each dot was visible for 26 ms at a position along

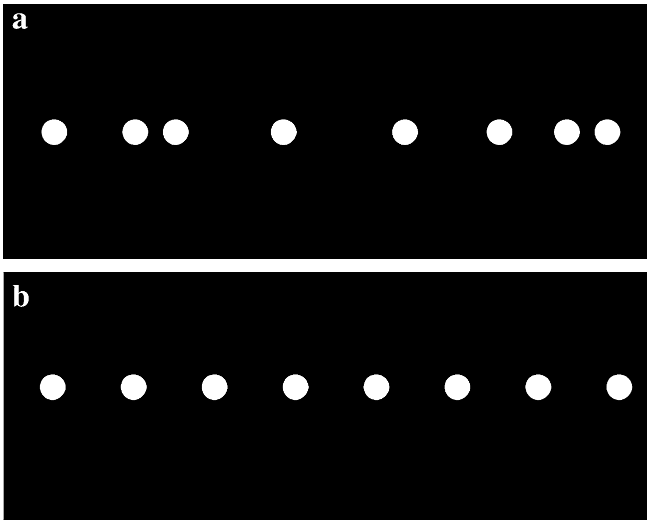

Fig. 1. Spatial position occupied by each dot of a target configuration. **a** Variable distances between consecutive dots, **b** constant distances between dots

the horizontal axis (presented sequentially from left to right), so that the eight dots formed a uni-dimensional spatial configuration (see Fig. 1).

The overall configuration covered 26.1 cm (8.7°), and the presentation sequence (of eight successive dots) lasted 3 s. Two experimental configurations were characterized by the spatial and/or the temporal ISI. These two configurations called "condition CV" and "condition VC", constant spatial ISIs were combined with variable temporal ISIt, and variable spatial ISIs were combined with constant temporal ISIt, respectively (see Fig. 2). These two conditions formed incongruent space-time configurations. Condition CV was presented to the first group which was asked to reproduce the spatial pattern of the target configuration, and condition VC, to the second group, which was asked to reproduce the temporal pattern.

Stimulus presentation and response recording were controlled by a dedicated application developed under the Labview 5.1 program. The measurement accuracy was about 1/100 cm. In the temporal task, participants' responses were obtained with a pushbutton connected to the computer's parallel port via a cable insulated from electromagnetic disturbances. Its ergonomic features and reliability (button sensitivity, lack of rebound of the signal provided by the switch, efficient anatomical position of the participant's hand) were designed to avoid all interference effects.

2.1.3 Procedure There were three phases in each experimental condition: a familiarization phase, a learning phase and a reproduction phase.

- Familiarization phase: In the familiarization phase participants acquainted themselves with the experimental apparatus, learning how to handle the mouse (the pushbutton) to situate the eight dots or to use the pushbutton to reproduce the rhythm.
- Learning phase: The participants' spatial task was to memorize each dot's location on the screen (i.e., the spatial configuration), whereas the participants' temporal task was to memorize the rhythm of dots appearance of the target configuration. In order to avoid all possible parasitic effects stemming from motor involvement, during the learning phase the participants did not reproduce the sequence. This phase thus consisted of 20 successive presentations of the same space-time sequence. Two consecutive sequences were separated by a 1.5 s time interval. Thus, the learning phase lasted 90 s. For each participant, the order of the conditions' presentation was counterbalanced so as to avoid order effects.
- Reproduction phase: At the end of the learning phase, the target configuration disappeared and the participant's task was to reproduce the learned pattern (spatial or temporal) from memory as precisely as possible. The participant had to situate each of the eight dots on the screen using the mouse (or to reproduce the rhythm using the pushbutton). Sixty trials were performed, without re-examination of the target. At the end of each reproduction, the next trial began automatically. In spatial task, no vertical precision was required with dots being automatically projected on the horizontal axis in the middle of the screen. In order to avoid possible influences of one condition on the next and possible effects of fatigue, the participants performed only one condition per day. The experiment took approximately 1 h in the spatial task, and approximately 20 min in the temporal task.

2.1.4 Data analysis From the data, a measure was used to examine the participants' reaction to temporal incongruence (in the spatial task) or the participants' reaction to spatial incongruence (in the temporal task). This analysis was done on particular distances or durations – short, baseline and long – aimed at studying the influence of temporal incongruence on the encoding of spatial characteristics (i.e., tau effect) and the influence of spatial incongruence on the encoding of temporal characteristics (i.e., kappa effect). The significance level (p) for the ANOVA statistical analysis (expressed by the Fisher's F) was set to *0.01*.

2.2 Experimental results

2.2.1 Analysis of the reaction to temporal incongruence: identification of the tau effect The question addressed in this section concerns the identification of the tau effect analysing the role of temporal context on the encoding of the configuration's spatial characteristics.

The reaction to the temporal incongruence was analyzed by comparing the distances produced in each condition. To this end, the seven distances were clustered into three groups, corresponding to the short, medium and long distances corresponding to the temporal ISIt of target configuration. The results showed (see Fig. 3) the constant distances of the target configuration were not reproduced as such [F (2, 118) = 7285, $p < 0.01$], with the distances covered at a faster speed (19.5 cm s^{-1} and 18.7 cm s^{-1}) being shortened and the distances covered at a slower speed (0.69 cm s^{-1} and 0.67 cm s^{-1}) being lengthened. Temporal

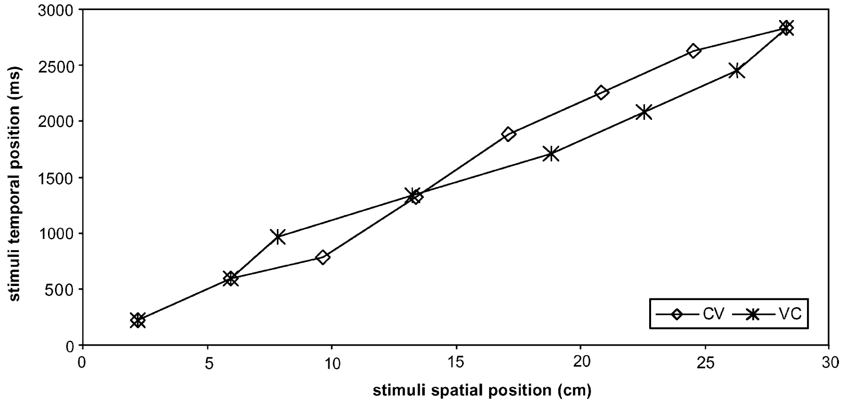

Fig. 2. Graphical representation of targets space-time configuration, plotting spatial (X axis) versus temporal (Y axis) positions of the stimuli. Note that, in this figure, slope which characterizes each ISI of the curves is the equivalent of speed in the text. When the slope is equal to 1, that we called "space-time baseline", the speed that covers the spatial ISIs is constant. However, when the slope is higher or lower than 1, the speed that covers the spatial ISIs is, respectively, faster or slower than space-time baseline. In condition CV, three of the (constant) spatial ISIs were characterized by a space-time baseline slope (i.e., 1.00 cm s^{-1}), two were characterized by a slope almost twice as high (1.95 and 1.87) and the other two by a slope about 1.3 times as low (0.69 and 0.67). In condition VC, three of the spatial ISIs were always characterized by a space-time baseline slope, the two larger spatial ISIs by a slope about 1.5 times higher (1.45 and 1.50) and the two smaller spatial ISIs by a slope twice as low (0.51 and 0.54)

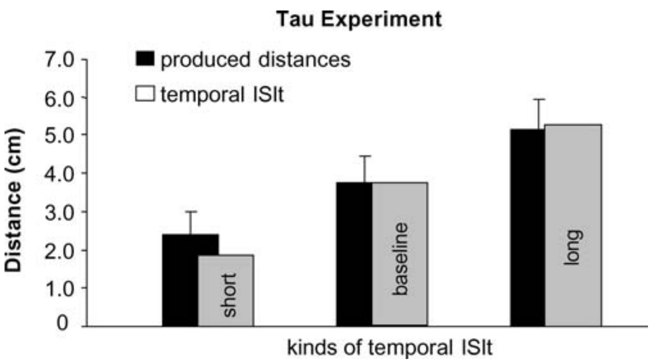

Fig. 3. Produced distances (in cm) and temporal ISIt (in ms). This experimental condition was characterized by one kind of spatial ISIs and three kinds of temporal ISIt. The reaction to the temporal disturbance was analyzed by comparing the distances produced. To this end, the seven distances were clustered into three groups, corresponding to the short, baseline and long durations of the target configuration. Figure 3 shows that the constant distances of the target configuration were not reproduced as such, with the distances covered with a short duration being shortened and the distances covered with a long duration being lengthened, so that a tau effect was identified

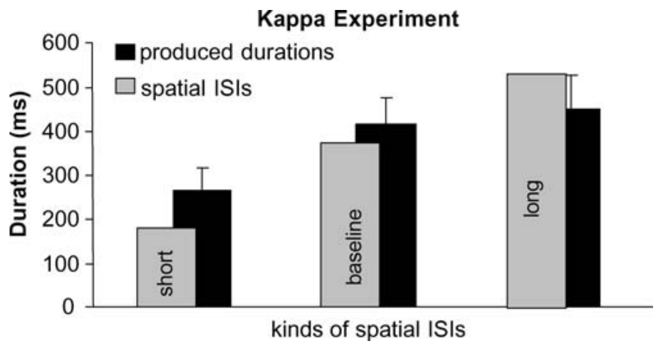

Fig. 4. Produced durations (in ms) and spatial ISIs (in cm). This experimental condition was characterized by three kinds of spatial ISIs and one kind of temporal ISIt. The reaction to the spatial disturbance was analyzed by comparing the durations produced. To this end, the seven durations were clustered into three groups, corresponding to the short, baseline and long distances of the target configuration. Figure 4 shows that the constant durations of the target configuration were not reproduced as such. In spite of the target characteristics, short, baseline, and long durations were produced, so that a kappa effect was found

incongruence thus profoundly influenced the encoding of spatial characteristics in this condition, so that a tau effect was identified.

2.2.2 Analysis of the reaction to spatial incongruence: identification of the kappa effect Along the same logic as that of the preceding experiment, the idea here was to identify a kappa effect. Thus, we studied the role of the spatial context on the encoding of temporal characteristics by measuring the different kinds of durations produced. Overall, the results showed (see Fig. 4) that the kinds of durations (i.e., short, medium, and long) produced by participants in the conditions VC differed significantly [$F(2, 118) = 11887.22$, $p < 0.01$]. In other words, the participant's task was to reproduce equitemporality. Notwithstanding the target characteristics, short, medium, and long durations were produced. Spatial incongruence clearly had an effect on the encoding of temporal information, so that a kappa effect was found.

Moreover, this reaction to spatial incongruence came with a disorganization of the target pattern (see Fig. 5). Looking in particular at the productions of short and long durations, we can see that short durations did not necessarily correspond to short distances [χ^2 (2) = 2.08, $p <$ 0.01] and that long durations did not necessarily correspond to long distances [χ^2 (2) = 4.08, $p < 0.01$].

3 The Model

3.1 Notations and definitions

Some notations and definitions will be convenient in describing the models and experiments. We note

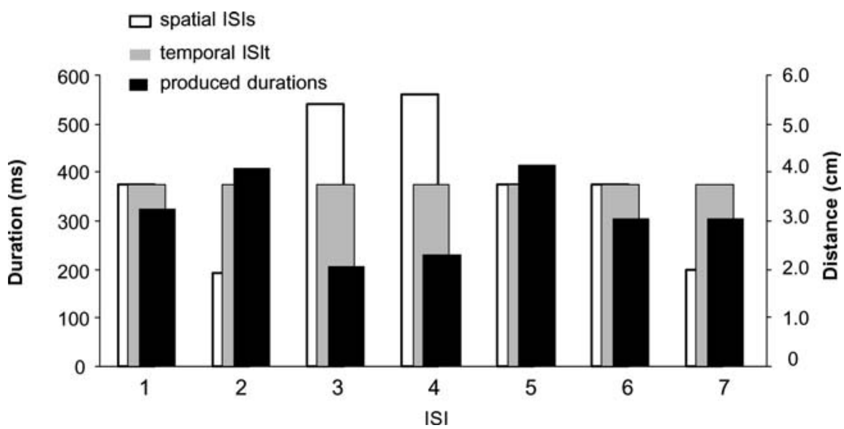

Fig. 5. Spatial ISIs, temporal ISIt, and produced durations by a participant in the kappa experiment, showing the disorganization of the pattern

$s_1, s_2, \ldots, s_n$ the successive spatial positions and $t_1, t_2, \ldots, t_n$ the respective times. In the present experiment, we have $n = 8$. We add the subscribe "exp" when we refer to the experiment i.e. the stimuli, and the subscribe "par" is used for the response of participants, i.e. s_1^{exp} is the ith position in the experiment and s_i^{par} is the position of ith dot locates by a participant. A notation without subscribe denotes the response of the model. It is also convenient to introduce the interval rather than the position and we note $\bar{s}_1, \bar{s}_2, \ldots, \bar{s}_n$ the spatial extents and $\bar{t}_1, \bar{t}_2, \ldots, \bar{t}_n$ the temporal intervals. We have $\bar{s}_i = s_i - s_{i-1}$ for $i = 1 \ldots n$ and we set $\bar{s}_0 = 0$ and $s_0 = 0$. Similar relations hold for the temporal durations. When we refer indifferently to the spatial or the temporal position, we note $X = (x_1, x_2, \ldots, x_n)$ where a capital letter is used for a vector. The data are normalized with respect to the maximal value of the total distance (and duration) of the experiment, i.e. we have $0 \leq x_1^{\text{exp}} < x_2^{\text{exp}} < \cdots < x_n^{\text{exp}} \leq 1$.

3.2 Modeling strategy and assumptions

In the tau and kappa experiments, the contextual information induces a stimulus deformation in the participant's reproductions of spatial or temporal extents. In the tau experiment, we assume that the participant's spatial reproduction results from a combination of the given distance, i.e. the stimuli, and a perceived distance based on an apparent velocity. The argument can be generalized to the kappa effect. More formally, we consider that the subject's observed response is monitored by a *recall information*, noted I_r , that drives the response towards the experimental data, and a *contextual information*, I_c , that leads to an effect in the spatial or temporal reproduction. The term information is used to emphasize that we aim at modeling a recall task. The recall information could be seen as a "perfect" memory of the target configuration whereas the contextual information could be seen as a disturbance due to the effect of perception, and in particular to perception of motion. We define the recall information as the difference between the model's response and the experiment. We use the following measure of information

$$I_r(X) = \left\| X - X^{\text{exp}} \right\|^2 \tag{1}$$

where $\|.\|$ is a norm on R^n . We take $\|X\|^2 = \sum_{i=1}^n x_i^2$. The contextual information I_c formalizes the assumptions about the nature of the effect. We will define it further below. In functional notation, we can write the response of the model $X^* = (x_1^*, x_2^*, \ldots, x_n^*)$ as the minimum of the function

$$I(X) = I_r(X) + \rho I_c(X) \tag{2}$$

where ρ is a positive weight parameter that controls the strength of the deformation induced by I_c . The value of ρ gives a formal measure of the effect of the temporal or spatial context in the reproduction; a small value gives a response close to the experiment whereas a large value yields a response mainly determined by the contextual information. For a given participant, the weight parameter is taken so as to minimize the difference between the response of the model and the response of participant, i.e. we minimize $\|X^* - X^{\text{par}}\|$. Recall that X^* is a function of ρ and thus we define for each participant a ρ value that is

$$\rho = \arg\min \left(\left\| X^* - X^{\text{par}} \right\| \right) \tag{3}$$

where argmin is the value of the argument, ρ , for which the expression $\|X^*(\rho) - X^{\text{par}}\|$ attains its minimum value as ρ varies from 0 to ∞ . For convenience, we do not introduce a specific notation for the optimal value of ρ . Model (2)–(3) is a globally fit model in the sense that the n positions are evaluated through the entire configuration and adjusted using one free parameter. This approach differs from models that only predict the intervals rather than the positions. Moreover, we address the problem of modeling the reproduction from memory of complex sequences, i.e. a sequence of stimuli greater than 2 or 3. A recursive prediction of each interval is not suitable for large sequences modeling since the estimation of the weight may fluctuate from one interval to the other and the global error may accumulate leading to an inaccuracy of the model. We will further discuss the implications of our approach.

The contextual information I_c models the assumptions on the origin of the effects, i.e. the deformation of the stimuli. As we previously mentioned, a common explanation is based on the imputation of motion to the display. Thus, the effects are associated with expectation of apparent motion and the reproduction of distances or durations are formally related to each other by a functional relationship using the velocity. This notion is referred to as the imputed velocity hypothesis and we will consider two distinct formalizations. Both are based on an imputed velocity and allow for a functional relation between space and time. We first revisit the constant velocity hypothesis that is commonly used in modeling the tau and kappa effects. Following our formalism, the constant velocity hypothesis can be written using the contextual information

$$I_c(X) = \left\| V - V_{\text{moy}} \right\|^2$$

where V_{moy} is the vector where each coordinate is the mean velocity $V_{\text{moy}} = (v_{\text{moy}} \ldots , v_{\text{moy}})^t$ imputed from the experiment, i.e. $v_{\text{moy}} = 1/n \sum_{i=1}^{n} \bar{s}_i^{\text{exp}} / \bar{t}_i^{\text{exp}}$. The vector V is the apparent velocity of the dot on each interval (obtained from X and X^{exp}.)

The constant velocity hypothesis assumes a deformation driven by a mean velocity, i.e. each velocity obtained as the ratio of spatial and temporal interval is compared to a given value. However it is not clear how a mean velocity could be adjusted from the display and used as a global reference for each apparent velocity. It seems to be relevant to relax this hypothesis and rather than fixing the velocity to a given value we require small velocity variations. We refer this hypothesis as the minimum acceleration hypothesis. Rather than a minimization of the velocity with respect to a given value, we minimize the difference between two successive velocities. The minimum acceleration hypothesis can be written using the following contextual information

$$I_c(X) = \left\| \Delta V \right\|^2$$

where ΔV is the vector where each coordinate gives the variation of the velocity. To clarify the equations, we distinguish between the two effects.

The tau effect. We determine the spatial positions $S^* = (s_1^*, s_2^*, \ldots , s_n^*)$. In the constant velocity hypothesis, the model's response minimizes the function

$$I(s_1, \ldots , s_n) = \sum_{i=1}^{n} (s_i - s_i^{\text{exp}})^2 + \rho \sum_{i=2}^{n} (v_i - v_{\text{moy}})^2,$$

where v_i is the velocity obtained from the predicted distance and the related temporal duration of the experiment, i.e. $v_i = (s_i - s_{i-1}) / (t_i^{\text{exp}}) - t_{i-1}^{\text{exp}}$ or, equivalently, $v_i = \bar{s}_i / \bar{t}_i^{\text{exp}}$. The primary spatial interval is not meaningful and we remove v_1 from the expression of the information. We now turn to the minimum acceleration hypothesis. The information can be written as

$$I(s_1, \ldots , s_n) = \sum_{i=1}^{n} (s_i - s_i^{\text{exp}})^2 + \rho \sum_{i=2}^{n-1} (v_{i+1} - v_i)^2,$$

We will discuss in detail the implications of these hypotheses in the part 3.3.

The kappa effect. We predict the durations $T^* = (t_1^*, t_2^*, \ldots , t_n^*)$ that are obtained as the minimum of the information

$$I(t_1, \ldots , t_n) = \sum_{i=1}^{n} (t_i - t_i^{\text{exp}})^2 + \rho \sum_{i=2}^{n-1} (v_i - v_{\text{moy}})^2,$$

where the velocity is derived from the spatial context $v_i = (s_i^{\text{exp}} - s_{i-1}^{\text{exp}}) / (t_i - t_{i-1})$. It is straightforward to generalize the minimum acceleration hypothesis to the kappa effect.

3.3 The tau effect: model implications

As we will see later (Sect. 4), the model fails to reproduce the participant's reproductions in the kappa experiment and we do not provide further details of the kappa model. In the following discussion, we will clarify the implications of the tau model and exhibit the underlying organization of space and time. The precise relation between each predicted position will be derived.

The response of the model is given by the minimum of the information function I . Thus, we seek for S^* that satisfies $dI/ds_i = 0$. In the constant velocity hypothesis we calculate (see Appendix)

$$(I_d - \rho M_v) S^* = S^{\text{exp}} + \rho v_{\text{moy}} U, \tag{4}$$

where I_d is the identity matrix, M_v is a symmetric tridiagonal matrix and U a vector. Both depend only on the temporal intervals of the stimuli T^{exp} . If ρ^{-1} is not an eigenvalue of M_v , i.e. $Id - \rho Mv$ is invertible, then the minimum exists and is unique. The case where ρ^{-1} is an eigenvalue does not appear in the effective calculation of ρ and we do not address this problem. Matrix M_v provides the relationship between each s_k^* and its sparseness gives the deeper of the inter-dependence. Given an index k, s_k^* is adjusted from its two neighboring positions. More precisely, from (4), we have

$$S_k^* = s_k^{\text{exp}} - \frac{\rho}{\bar{t}_k^{\text{exp}}} \left(\frac{\bar{s}_k^*}{\bar{t}_k^{\text{exp}}} - v_{\text{moy}} \right) + \frac{\rho}{\bar{t}_k^{\text{exp}}} \left(\frac{\bar{s}_{k+1}^*}{\bar{t}_{k+1}^{\text{exp}}} - v_{\text{moy}} \right), \tag{5}$$

for $k = 2, \ldots , 7$. Note that (5) is an implicit expression that emphasizes the non trivial space-time dependencies implied by our model. An explicit relation could be derived for small ρ values. It is straightforward to calculate the first-order expansion

$$S^* = (I_d + \rho M_v) S^{\text{exp}} + \rho v_{\text{moy}} U,$$

that shows explicitly the deformation induced by the context. Conversely large values of ρ lead to a model with a purely constant velocity response. In fact, we solve $M_v S^* = v_{\text{moy}} U$ and we find

$$s_k^* = s_{k-1}^* + v_{\text{moy}} \bar{t}_k^{\text{exp}} \quad \text{for } k = 2, \ldots , n,$$

where s_1^* is arbitrarily chosen. Given s_1^* , the other positions of the model are adjusted sequentially in order to have a constant velocity response.

We now turn to the minimum acceleration hypothesis. The model gives a response s^* that satisfies

$$(I_d - \rho M_a) S^* = S^{\text{exp}},$$

where M_a is a bandmatrix of band width 5. Equation (6) shows the fundamental difference between the two hypotheses. The structure of the matrix M_a shows that the kth position of the dot is associated with the two pairs of adjacent positions, i.e. , $k-1, k-2$ and , $k+1, k+2$. The precise relation is given in appendix. As previously, small

ρ values lead to a model's response close to the stimuli. For large ρ values we solve $M_a S^* = 0$. From the observation that the dimension of the nullspace of M_a is two, the model's response depends on two arbitrary chosen parameters, let us take s_1^* and s_2^* . The minimum acceleration solution is given by

$$s_k^* = s_{k-1}^* + \bar{t}_k^{\text{exp}} / \bar{t}_{k-1}^{\text{exp}} \left(s_{k-1}^* - s_{k-2}^* \right),$$

where $k = 3, \ldots, n$. Taking the particular value $s_2^* = s_1^* + v_{\text{moy}} \bar{t}_1^{\text{exp}}$, we obtain the solution given by the constant velocity hypothesis. Thus, in the limit of large ρ , the constant velocity hypothesis could be seen as a specific case of the minimum acceleration hypothesis. Indeed a constant velocity leads to a null acceleration so that the acceleration reaches its minimum.

3.4 Accuracy of the model

Efficiency of a model is a trade off between the number of fitted parameters and the number of responses that could be described. In our modeling, we consider one free parameter for each participant: the weight parameter. To check the validity of the model, i.e. if the assumption on the produced effect is relevant, we compare the response of the model with the response of the participant. More precisely, we calculate for each participant the difference between the model and the participant's response, i.e. we calculate the normalized difference

$$D = 1/n \left\| X^{P^{**}} - X^{\text{par}} \right\|^2 = 1/n \sum_{i=1}^{n} (x_i^{**} - x_i^{\text{par}})^2$$

where X^{**} is the response of the model for ρ optimum. In particular, we have $\left\| X^{**} - X^{\text{par}} \right\| \leq \left\| X^{\text{exp}} - X^{\text{par}} \right\|$ since X^{exp} is the response of the model for the particular value $\rho = 0$. Since $|x_i^{\text{exp}} - x_i^{\text{par}}| \leq 1$ we get $1/n \left\| X^{\text{exp}} - X^{\text{par}} \right\| \leq 1$ and thus the difference between the participant response and the optimized (with respect to ρ) response of the model, measured by D, is lesser than 1. However, to have a meaningful measure of the accuracy of our model, we consider the error produced by a random prediction. We define the random variable $d^{\text{ran}} = 1/n \left\| X^{\text{ran}} - X^{\text{par}} \right\|$ where $X^{\text{ran}} = (x_1^{\text{ran}}, x_2^{\text{ran}}, \ldots, x_n^{\text{ran}})$ and x_i^{ran} are ordered random number between 0 and 1. By definition we have $0 \leq d^{\text{ran}} \leq 1$ and the higher value is reached when $x_i^{\text{ran}} = 1$ and $x_i^{\text{par}} = 0$ or conversely. The random variable d^{ran} is a measure of the prediction given by a random model. We therefore compare D with the expectation of d^{ran} and we note that $D^{\text{ran}} = E(d^{\text{ran}})$. Numerically, we find (not shown) that the expectation poorly depends on the participant's response and for simplicity we only consider the average over the participants. We calculate $D^{\text{ran}} = 0.02$. For the tau and kappa experiments, we will compare the difference D with the value obtained from a random prediction that is $D^{\text{ran}} = 0.02$.

4 Model results

The results presented in this section are numerical simulations of the model that we further compared with those of the experimental study.

4.1 The tau effect

In Table 1 the model predictions are compared with the participant responses in the tau experiment. For each participant, the error between the participant response and the constant velocity model prediction, on the one hand, and minimum acceleration model prediction, on the other hand, are presented. As explained above (i.e., in the Sect. *3.4. Accuracy of the model*), the difference between the model and the participant response is considered as our criteria to check the validity of the model and have to be compared with the error produced by the random model, i.e. 0.02. As shown in Table 1, the two models provide a result that is 20 times and 30 times better than the random model, respectively. Therefore, we argue that our approach captures the tau effect. As a first stage of the results analysis, we will give an interpretation of the weight parameter. We will further discuss our results through the predicted positions of the dots.

4.1.1 Tau weight parameter As we described above (Sect. 3.2.), ρ is a weight parameter that controls the strength of the deformation induced, in the circumstances of the tau effect, by the temporal context and is adjusted for each participant. Based on the hypothesis that space-time dependencies are organized according to an imputed velocity, this formal measure is a way to quantify the temporal effect on the spatial reproduction. The ρ value presented for each participant in Table 1 is obtained using the optimisation procedure that we have described before. A typical shape of the global difference between the model and the participant response is given in Fig. 6, for different values of the weight parameter. The minimum of the difference gives the optimised ρ value. The quadratic profile of the curve indicates that the determination of an optimal ρ is well-posed. Recall that for $\rho = 0$ the curve gives the difference between the participant response and the experiment whereas for $\rho = +\infty$, we calculate the difference with the purely constant velocity (or minimum acceleration) model.

Results show that, on average, the predictions of the minimum acceleration model are about 30% more accurate than those of the constant velocity model. However, predictions for the participants 1, 7, 10 and 12 (see Table 1) are not significally improved by the minimum acceleration model. This result shows that the minimum acceleration hypothesis is not just a generalization of the constant velocity hypothesis but another formalization of the imputed motion hypothesis.

4.1.2 Model predictions Figure 7 presents the eight positions of the dots reproduced by the participants (Fig. 7a), and those predicted by the constant velocity model (Fig. 7b) or by the minimum acceleration model (Fig. 7c). The error between the predicted and the effective response is represented in Fig. 7d. In the constant velocity hypothesis, the error related to each spatial position is well fitted with the regression line $y = 0.0033x + 0.0089$ (Fig. 7d). This result indicates the existence of a linear relation between the error and the position. The quality of the fitting

Table 1. The quantification of the tau effect (through the weight parameter ρ) and the measure of the difference between participants' responses and model predictions in the constant velocity and minimum acceleration hypotheses. This difference has to be compared with the difference between participant's responses and the predictions given by a random model (see Sect. 3.4) that is D^ran (*10^4) = 200

participants	constant velocity hypothesis		minimum acceleration hypothesis	
	ρ (x 10^2)	error between participant and model responses (x 10^4)	ρ (x 10^2)	error between participant and model responses (x 10^4)
1	0.08	0.6	0.03	0.6
2	8.01	8.9	13.12	6.0
3	10.45	6.1	13.02	1.1
4	3.37	6.5	6.53	4.5
5	1.49	25.0	23.78	4.9
6	1.00	16.0	15.55	12.0
7	6.67	9.7	9.20	9.5
8	2.88	9.0	0.06	11.0
9	1.35	2.2	1.74	1.6
10	18.67	9.2	18.90	9.4
11	8.31	4.9	14.53	3.2
12	3.63	10.0	1.17	10.0
average	**5.49**	**9.0**	**9.80**	**6.1**
SD*	***5.39***	***6.4***	***7.95***	***4.0***

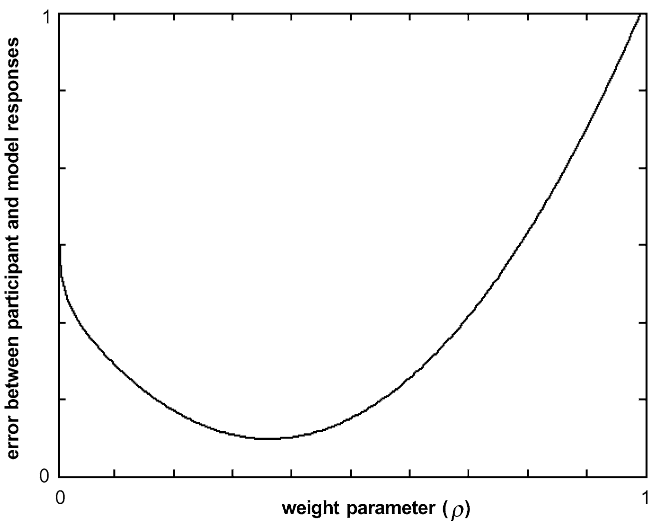

Fig. 6. The typical shape of the difference between the response predicted by the model and the participant response in the tau experiment. The ρ value that gives the minimum of the difference gives the quantification of the tau effect for a given participant. The quadratic profile indicates that optimisation problem for the quantification of the tau effect is well-posed

is evaluated with a determination coefficient (R^2) of 0.8. At first sight, it seems that the linearly increasing error in position is the result of a phenomenon of accumulating errors. In the minimum acceleration model, the situation is different. Indeed, results show that the errors are fitted with a binomial tendency curve whose equation is $y = -0.006x^2 + 0.0057x + 0.01$ and whose R^2 is 0.7 (see Fig 7d). The Gaussian profile of the curve illustrates a more homogeneous repartition of the error over the spatial positions. This phenomenon of the error's homogenisation observed in the minimum acceleration model could be explained by the interrelations between each predicted dot positions. The model implications have shown that a given position is related to its four neighbours in the minimum acceleration model whereas interrelations are made between its two neighbours in the constant velocity model. These interrelations introduce a deeper dependence between the dots leading to a better accuracy of the model.

Figure 8 presents the seven spatial intervals between two successive dots reproduced by the participants (Fig. 8a), and predicted by the constant velocity model (Fig. 8b) or by the minimum acceleration model (Fig. 8c). It is interesting to note (Fig. 8d) that on average, the error associated to each spatial interval predicted by the two models does not significantly differ (0.019 and 0.018, respectively). So, this result shows clearly that the phenomenon of accumulating errors, previously observed in positions, disappears when we analyse spatial intervals. Regarding the errors on the predicted spatial positions (Fig. 7d), we argue that a compensation (not accumulating) of the errors on the spatial positions occurs in the constant velocity model. However, it is unclear if this compensation is related to the model or to the specific temporal context used in the experiment. Further experiments will be desirable to conclude on this point. Nevertheless, as the action system to drive pointing movements (Van Wieringen and Beek 1997), these results obviously show that distance and position are two complementary information that allow to better explore the implications of imputed motion models. The choice of fitting spatial positions or spatial intervals might strongly influence the capabilities of a model, but our model works with spatial positions since they are the quantities reproduced by the participants. Moreover, if the spatial intervals analysis allows to explain the phenomenon of error compensation obtained with the constant velocity hypothesis (see Fig. 8d), our choice accounts for the reported results, especially for the error homogenization observed with the minimum acceleration hypothesis (see Fig. 7d).

4.2 The kappa effect

For each participant, we calculate the difference between the temporal response and the model prediction. In the

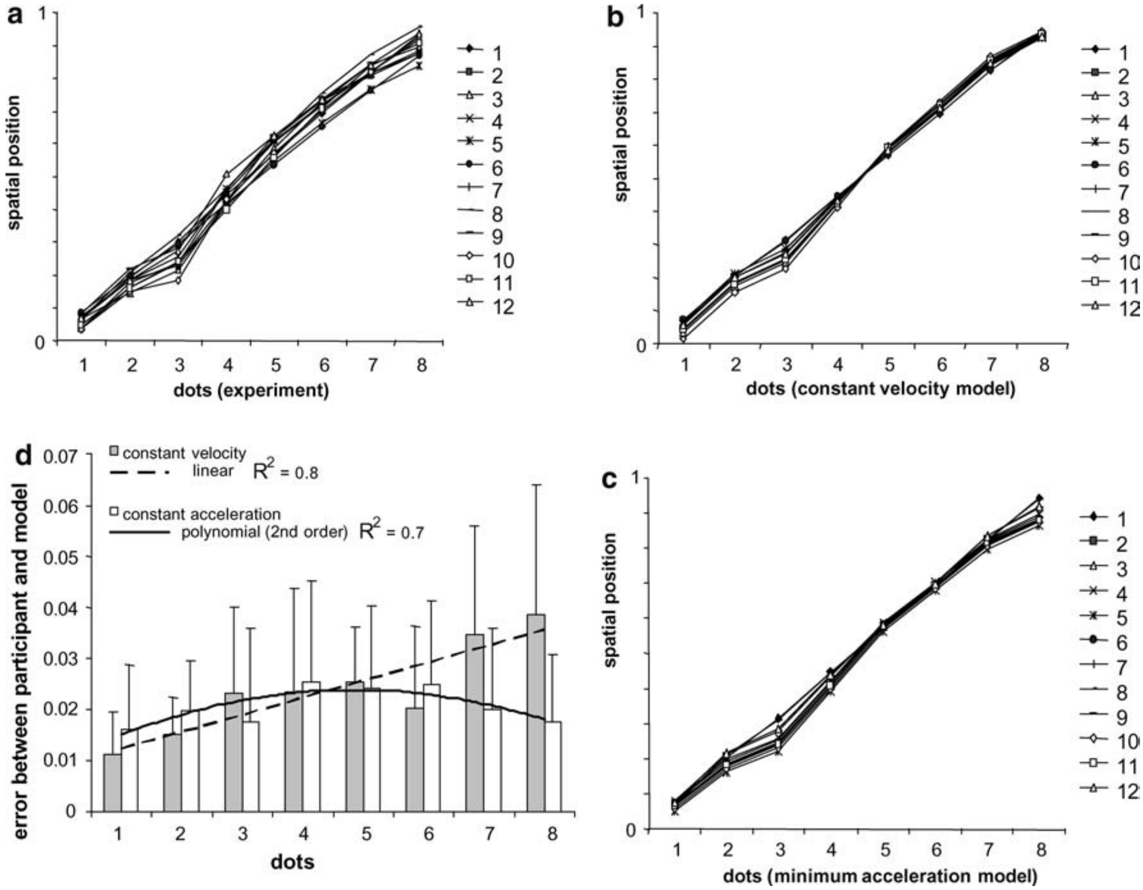

Fig. 7. The eight spatial positions of the dots **a** reproduced by the participants, and **b** predicted by the constant velocity model, or **c** by the minimum acceleration model. Panel **d** shows the accuracy of the predictions provided by the constant velocity model (*in light grey*) and the minimum acceleration model (*in white*), for each spatial position. In the constant velocity hypothesis, the linearly increasing error in positions, which translates the existence of a linear relation between the error and the position, shows a phenomenon of accumulating errors. However, in the minimum acceleration model, the Gaussian profile of the curve, which points out a more homogeneous repartition of the error over the spatial positions, can be explained by the interrelations between each predicted dot position to its four neighbours in the minimum acceleration model whereas interrelations are made between its two neighbours in the constant velocity model. These interrelations introduce a deeper dependence between the dots leading to a better accuracy of the model

constant velocity hypothesis, we observe an average error of 0.01 (Table 2) that is not significally different to that of the random model (0.02). A similar result holds for the minimum acceleration hypothesis. At first sight, we conclude that the participant temporal responses are not explained by an imputed motion from the display. Even if a small effect is captured (since we obtain a nonzero value for ρ), we cannot attribute a meaningful interpretation of the weight parameter.

The shape of the error function (Fig.. 9) reveals clearly that the kappa effect is not characterized by our model since a nonzero minimum fails to be found.

A tentative version of an ordinal model (not shown) has been investigated. To improve the model, we introduced a permutation that associated the velocity i with a duration j where the subsripts i and j could be different. In other words, the functional dependence is done on pairing the ith velocity and the jth duration where $j = p(i)$ and p is some optimized permutation. However, this ordinal model fails to reproduce the observed data.

5 Discussion

In this article the fundamental question of space and time dependencies in the reproduction of spatial or temporal extents is studied. We focused on the effects of variable durations and distances, respectively, when reproducing constant distances (tau effect) or durations (kappa effect). As a starting point, we considered the Sarrazin et al. study (2004). In their experiments, the authors demonstrated that, with visual patterns, tau and kappa effects also occurred in a memory reproduction task using complex space-time patterns. Their results showed participants were presented with a pattern defined by a constant spatial ISIs associated with a variable temporal ISIt, spatial responses revealed a strong dimensional interference effect (the tau effect). The response pattern was not only modified but completely transformed, as participants produced spatial ISIs which corresponded to the presented temporal ISIt. In a similar way, when participants were presented with constant temporal ISIt associated with variable

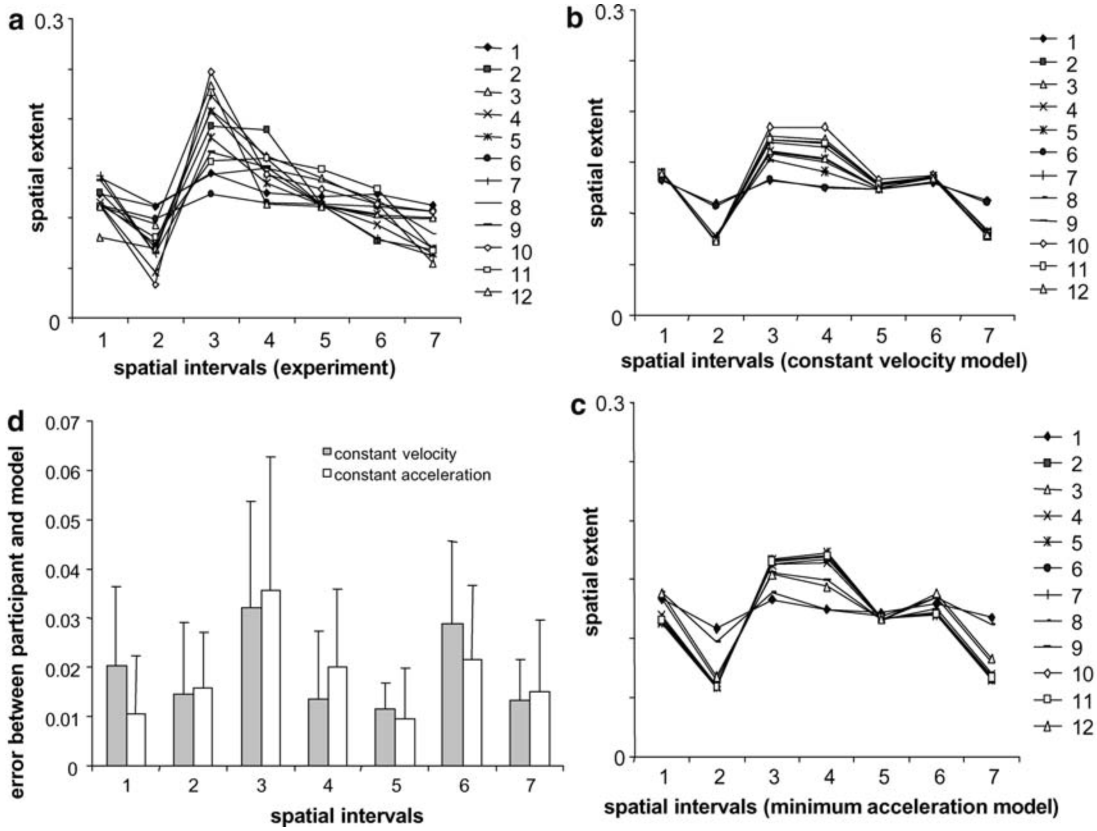

Fig. 8. The seven spatial intervals between two successive dots **a** reproduced by the participants, and **b** predicted by the constant velocity model, or **c** by the minimum acceleration model. Panel **d** shows the accuracy of the predictions provided by the constant velocity model (*in light grey*) and the minimum acceleration model (*in white*), for each spatial interval. Note that the error associated to each spatial interval predicted by the two models does not significantly differ. This result shows clearly that the phenomenon of accumulating errors, previously observed in positions, disappears when we analyze spatial intervals and let us to argue that a compensation (not accumulating) of the errors on the spatial positions occurs in the constant velocity model

spatial ISIs and the requested response was temporal, they observed a transformation of the response that led to the production of variable temporal ISIt (the kappa effect). However, even though the kappa effect influences the temporal characteristics of the response, this influence was no t equivalent to the tau effect, as earlier perceptual experiments have already shown (Geldard 1972; Jones and Huang 1982).

Different hypotheses have been suggested on the origin of the tau and kappa effects. We assumed that a mathematical model can offer a criticism of these assumptions and reveal the underlying mechanism of these effects. Based on the imputed velocity hypothesis we provided a globally fit model that addressed the question of large sequence modelling. Our model is driven by a recall force derived from the difference between the response and the stimuli and a deformation force that is induced by the context. The salience of the context is measured in a weight parameter that is fitted in order that the model reproduced the response of participants. Using the constant velocity hypothesis, we showed that our model is capable of producing the behavior observed in the tau experiment. Moreover, we improved the accuracy of the model introducing a new hypothesis based on small velocities variation. This last result sets the problem of acceleration on perception. However, studies on motor skills in general,

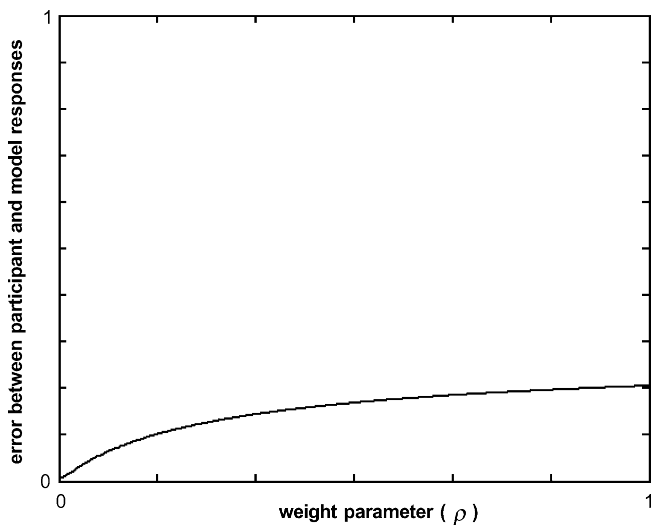

Fig. 9. The profile of the difference between the participant response (in the kappa experiment) and the model response in the constant velocity hypothesis as a function of ρ . The non-quadratic shape of the function indicates that the kappa effect is not captured by our model

and on trajectory formation in particular, have assumed that a major goal of motor coordination is the production of the smoothest possible movement of the effectors, e.g., the hand (Flash and Hogan 1985), minimizing the rate of change of acceleration, i.e., the minimum-jerk (Hogan 1984). The minimum-jerk model predicts the smoothest trajectory for a class of human movements and so provides us with kinematics measurement of skilled motor performance (Dingwell et al. 2004). The minimum acceleration model follows this argument in the sense that we homogenized the velocity appearance of the percept through a minimization of the velocity variation. This similitude between the minimum acceleration hypothesis (developed starting from an imputed motion hypothesis which is issue from the perception field) and the minimum-jerk model (which results from the motor domain) allows to think that general principles rule both perception (and perceptive memory) and motor skills. This idea finds all the more echo as motor theories of perception suppose particularly that trajectory formation principles govern perception as well, when Viviani and Stucchi (1992) assumed "*that the process of perceptual selection is constrained or guided by motor schemes, that is, by procedural, implicit knowledge that the central nervous system has with regard to the movements it is capable of producing. (p. 603)*".

However, our results have shown that the imputed velocity hypothesis fails to reproduce the data obtained in the kappa experiment. In other words, kappa experiment incites to spatio-temporal perception what cannot be explained by motor theories of perception. Thus, the fact that (1) tau effect is explained by imputed motion models, (2) process of selective perception is governed by movement trajectory formation principles, and (3) kappa effect cannot be modelled by an imputed motion hypothesis, allow supposing that kappa experiment does not agree with the perception of a physical event. In other words, we hypothesized that kappa experiment leads to the perception of an artificial event that is not ruled by trajectory formation principles. Of course, other studies are needed to verify this hypothesis. Nevertheless, this result definitively shows that these effects are not symmetric. The kappa effect has been commonly regarded as the converse phenomenon of the tau effect but, as Shigeno (1993) emphasized, both effects do not follow the same psychological process. The kappa effect is characterized by a modification of the target sequence order reported as a disorganization of the target sequence (Sarrazin et al. 2004), i.e. the participants produced variable durations which did not correspond to the related perceived distances. Since (1) the tau experiment has the same complexity as the kappa experiment (seven intervals) and (2) the tau effect is not characterized by a disorganization of the target configuration, it seems that the disorganization of the target sequence reflects the main difference between the two effects. Those asymmetric phenomenons clearly set an unresolved and difficult question about the relation between space and time. Classically, the kappa effect has been studied with the paradigm of two successive space-time intervals and, in this context; the disorganization of the target sequence was completely ignored. Obviously, the absence of disorganization was directly dependent of the experimental situation since the contextual effect of an interval was analysed across the judgment of the other. Thus it seems crucial (1) to redefine the kappa effect as a phenomenon in which the contextual influence and the modification of events order are associated and (2) to study this effect across the reproduction of sequences constituted of more than two space-time intervals. Without being able, at the present time, to propose an explanation for this effect, it seems that this relation differs accordingly as we have the participant to pay attention upon the spatial or temporal dimension when it is disturbed by a temporal or spatial context.

6 Appendix

We consider the model of the tau experiment. We distinguish between the two following hypotheses:
The constant velocity hypothesis
For a given weight parameter ρ , the response of the model minimizes the information $I(s_1,s_2,\dots,s_n)$. Thus the successive positions are given solving $\nabla I = 0$ where ∇I is the vector of the partial derivatives of I .We calculate the following partial derivatives of I for the tau model:

$$\frac{dI}{ds_1} = 2(s_1 - s_1^{\exp}) - \frac{2\rho}{\bar{t}_2^{\exp}(v_2 - v_{moy})},$$
$$\frac{dI}{ds_k} = 2(s_k - s_k^{\exp}) + \frac{2\rho}{\bar{t}_k^{\exp}(v_k - v_{moy})} - 2\rho/\bar{t}_{k+1}^{\exp}(v_{k+1} - v_{moy}), \quad \text{for } k = 2, \dots, n-1,$$
$$\frac{dI}{ds_n} = 2(s_n - s_n^{\exp}) + \frac{2\rho}{\bar{t}_n^{\exp}(v_n - v_{moy})}.$$

The successive positions of the model $(s_1^*, s_2^*, \dots, s_n^*))$ are given solving $dI/ds_k = 0$. Using $v_2 = (s_2 - s_1)/\bar{t}_2^{\exp}$ the first equation, obtained for $k = 1$, could be written as

$$s_1^* - s_1^{\exp} - \frac{\rho(s_2^* - s_1^* - v_{moy}\bar{t}_2^{\exp})}{(\bar{t}_2^{\exp})^2} = 0$$

leading to $(1+\rho/(\bar{t}_2^{\exp})^2)s_1^* - \rho/(\bar{t}_2^{\exp})^2 s_2^* = s_1^{\exp} + \rho v_{\mathrm{moy}}/\bar{t}_2^{\exp}$. Using $v_k = (s_k - s_{k-1})/\bar{t}_k^{\exp}$, similar calculations can be performed for $k = 2, \dots, n$. Finally, we obtain the system of equations $(I_d - \rho M_v)S^* = S^{\exp} + \rho v_{\mathrm{moy}} U$ where I_d is the identity matrix,

$$M_v = \begin{pmatrix} -t_2^{-2} & t_2^{-2} & 0 & \cdots & 0 \\ t_2^{-2} & -t_2^{-2} - t_3^{-2} & t_3^{-2} & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & t_{n-1}^{-2} & -t_{n-1}^{-2} - t_n^{-2} & t_n^{-2} \\ 0 & \cdots & 0 & t_n^{-2} & -t_n^{-2} \end{pmatrix},$$

and $U = (-t_2^{-1} \quad t_2^{-1} - t_3^{-1}, \dots, t_{n-1}^{-1} - t_n^{-1} \quad t_n^{-1})^t$. In U and M_v, for convenience we use "t" instead of "$\bar{t}^{\exp}$" the temporal duration of the stimuli. *The minimum acceleration hypothesis*

Table 2. The quantification of the kappa effect (through the weight parameter ρ) and the measure of the difference between participants' temporal positions responses and constant velocity model predictions of the corresponding temporal positions. This difference has to be compared with the difference between partipant's responses and the predictions given by a random model (see Sect. 3.4) that is Dran (*10^2) = 2

participants	ρ (x 10^4)	error (x 10^2)
1	0.66	0.77
2	6.19	0.27
3	12.00	0.16
4	12.00	0.08
5	0.66	0.37
6	0.66	0.40
7	0.66	0.26
8	3.78	0.60
9	13.00	1.35
10	25.00	0.61
11	40.00	7.03
12	0.66	1.34
average	**9.61**	**1.10**
***SD* (1)**	***12.17***	***1.91***

The parital derivatives of I are given by

$$\frac{dI}{\mathrm{d}s_1} = 2(s_1 - s_1^{\exp}) + \frac{2\rho}{\bar{t}_2^{\exp}(v_3 - v_2)}$$

$$\frac{dI}{\mathrm{d}s_2} = 2(s_2 - s_2^{\exp}) + 2\rho/(1/t_3^{-\exp})(v_3 - v_2) + 2\rho/t_3^{-\exp}(v_4 - v_3)$$

$$\begin{aligned} dI/\mathrm{d}s_k &= 2(s_k - s_k^{\exp}) + 2\rho/t_k^{-\exp}(v_k - v_{k-1}) \\ &\quad -2\rho/(1/t_{k+1}^{-\exp})(v_{k+1} - v_k) \\ &\quad +2\rho/t_{k+1}^{-\exp}(v_{k+2} - v_{k+1}) \\ &\quad \text{for } k = 3, \ldots, n-2 \\ dI/\mathrm{d}s_{n-1} &= 2(s_{n-1} - s_{n-1}^{\exp}) + 2\rho/t_{n-1}^{-\exp}(v_{n-1} - v_{n-2}) \\ &\quad -2\rho/(1/t_n^{-\exp} + 1/t_{n-1}^{\exp})(v_n - v_{n-1}) \\ dI/\mathrm{d}s_n &= 2(s_n - s_n^{\exp}) + 2\rho/t_n^{-\exp}(v_n - v_{n-1}) \end{aligned}$$

We obtain the system $(I_d - \rho M_a)S^* = S^{\exp}$ where the matrix M_a is given by

$$\begin{pmatrix} -t_2^{-2} & (t_2t_3)^{-1}+t_2^{2} & -(t_3t_2)^{-1} & 0 & \cdots & \cdots & 0 \\ (t_3t_2)^{-1}+t_2^{-2} & -t_2^{-2}-2(t_3t_2)^{-1}-2t_3^{-2} & 2t_3^{-2}+(t_3t_2)^{-1}+(t_4t_3)^{-1} & -(t_4t_3)^{-1} & \ddots & & \vdots \\ -(t_3t_2)^{-1} & 2t_3^{-2}+(t_3t_2)^{-1}+(t_4t_3)^{-1} & -2t_3^{-2}-2(t_4t_3)^{-1}-2(t_4)^{-2} & 2t_4^{-2}+(t_4t_3)^{-1}+(t_5t_4)^{-1} & -(t_5t_4)^{-1} & & \vdots \\ 0 & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ & & -(t_{n-3}t_{n-2})^{-1} & 2t_{n-2}+(t_{n-3}t_{n-2})^{-1}+(t_{n-2}t_{n-1})^{-1} & -2t_{n-2}^{-2}-2(t_{n-2}t_{n-1})^{-1}-2t_{n-1}^{-2} & 2t_{n-1}^{-2}(t_{n-2}t_{n-1})^{-1}+(t_{n-1}t_n)^{-1} & -(t_{n-1}t_n)^{-1} \\ \vdots & & \ddots & -(t_{n-2}t_{n-1})^{-1} & 2t_{n-1}^{2}+(t_{n-2}t_{n-1})^{-1}+(t_{n-1}t_n)^{-1} \; -(t_nt_{n-1})^{-1} & -2t_{n-1}^{-2}-(t_{n-1}t_n)^{-1}-2t_n^{-2} \; t_n^{-2}(t_nt_{n-1})^{-1} & -t_n^{-2}+(t_{n-1}t_n)^{-1} \; -t_n^{-2} \\ 0 & \cdots & \cdots & 0 & & & \end{pmatrix}$$

As previously, "t" is used for the abbreviation of "$\bar{t}^{\exp}$".

Acknowledgements. This research was supported by the Institut National de la Recherche en Informatique et en Automatique (INRIA).

References

Abe S (1935) Experimental study on the co-relation between space and time. Tohoku Psychol Folia 3:53–68

Anderson NH (1974) Algebric models in perception. In: Carterette EC, Friedman MP (eds) Handbook of perception vol 2. Academic, New York

Benussi V (1913) Versuche zur Analyse taktil erweckter Scheinbewegungen. Archiv für die gesamte Psychologie 36:58–135

Bill JC, Teft LW (1969) Space-time relations: effects of time on perceived visual extend. J Exp Psychol 81:196–199

Cohen J, Hansel CEM, Sylvester JD (1953) A new phenomenon in time judgement. Nature 172(4385): 901

Cohen J, Hansel CEM, Sylvester JD (1955) Interdependence in judgments of space, time, and movement. Acta Psychol 11:360–372

Collyer CE (1977) Discrimination of spatial and temporal intervals defined by three light flashes: effects of spacing on temporal judgments. Percep Psychophys 21:357–364

Dingwell JB, Mah CD, Mussa-Ivaldi FA (2004) Experimentally confirmed mathematical model for human control of a non-rigid object. J Neurophysiol 91:1158–1170

Flash T, Hogan N (1985) The coordination of arm movements: an experimentally confirmed mathematical model. J Neurosci 5:1688–1703

Finke RA, Shepard RN (1986) Visual functions of mental imagery. In: Boff KR, Kaufman L, Thomas JP (eds) Handbook of perception and human performance, vol.II. cognitive processes and performance. Wiley, New York

Fitts PM (1954) The information capacity of the human motor system in controlling the amplitude of movement. J Exp Psy 47:381–391

Geldard FA (1972) The human senses (2nd ed) Wiley, New York

Gibson JJ (1966) The problem of temporal order in stimulation and perception. J Psychol 62:141–149

Helson H (1930) The tau effect: an example of psychological relativity. Sci 71:536–537

Helson H, King SM (1931) The tau effect: an example of psychological relativity. J Exp Psychol 14:202–217

Hogan N (1984) An organizing principle for a class of voluntary movements. J Neurosci 4:2745–2754

Jones B, Huang YL (1982) Space-time dependencies in psychological judgment of extent and duration: algebric model of the tau and kappa effects. Psychol Bull 91:128–142

Kelso JAS (1981) On the oscillatory basis of movement. Bull Psychon Soc 18, 63

Lechelt EC, Bochert R (1977) The interdependence of time and space in somesthesis: the tau effect re-examined. Bull Psychon Soc 10:191–193

Lee D (2000) Learning of spatial and temporal patterns in sequential hand movements. Cogn Brain Res 9:35–39

Mashour M (1964) Psychophysical relations in the perception of velocity. Almquist and Wiksell, Stockholm

Meyer DE, Abrams RA, Kornblum S, Wright CE, Smith JEK (1988) Optimality in human motor performance: ideal control of rapid aimed movements. Psychol Rev 95(3):340–370

Mottet D, Bootsma RJ (1999) The dynamics of goal-directed rhythmical aiming. Biol Cybern 80:235–245

Price-Williams DR (1954) The kappa effect. Nature 173:363–364

Sarrazin JC, Giraudo MD, Pailhous J, Bootsma RJ (2004) Tau and kappa effects revisited: dynamics of balancing space and time in memory. J Exp Psychol Hum Perception Perform 30(3):411–430

Schill K, Zetzsche C (1995) A model of visual spatio-temporal memory: the icon revisited. Psychol Res 57:88–102

Shigeno S (1993) The interdependence of pitch and temporal judgments by absolute pitch possessors. Percept Psychophys 54(5):682–692

Temprado JJ, Chardenon A, Laurent M (2001) Interplay biomechanical and neuromuscular constraints on pattern stability and attentional demands in a bimanual coordination task. Neurosci Lett 303:127–131

Van Wieringen PCW, Beek PJ (1997) Distance versus position information in the control of aiming movements. Behav Brain Sci 20:323–324

Viviani P, Stucchi N (1992) Biological movements look uniform: evidence of motor-perceptual interactions. J Exp Psychol Hum Percept Perform 18:603–623

Zanone PG, Kelso JAS (1992) Evolution of behavioral attractors with learning – nonequilibrium phase-transitions. J Exp Psychol Hum Percept Perform 18:403–421