Time integration of nonsmooth mechanical systems with unilateral contact. Conservation and stability of position and velocity constraints in discrete time.

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NonSmooth Multibody Systems
Scleronomous holonomic perfect unilateral constraints

\[
\begin{aligned}
M(q(t))\dot{v} &= F(t, q(t), v(t)) + G(q(t))\lambda(t), \text{ a.e} \\
\dot{q}(t) &= v(t), \\
g(t) &= g(q(t)), \quad \dot{g}(t) = G^T(q(t))v(t), \\
0 &\leq g(t) \perp \lambda(t) \geq 0, \\
\dot{g}^+(t) &= -e\dot{g}^-(t),
\end{aligned}
\]  

(1)

where \(G(q) = \nabla g(q)\) and \(e\) is the coefficient of restitution.

Unilateral constraints (unilateral contact, Signorini condition)
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Introduction & Motivations
Problem setting
Objectives & means
Outline
A naive projection scheme
A projection/activation scheme
Conclusions & Perspectives

NonSmooth Multibody Systems (NSMBS)

Academic examples

(a) The bouncing ball
(b) The linear oscillator
(c) The chain of balls
(d) The rocking block

Figure: Simple archetypal test examples
NonSmooth Multibody Systems (NSMBS)

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Figure: Analytical solutions.
Mechanical systems with contact, impact and friction

Simulation of Circuit breakers (INRIA/Schneider Electric)
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Mechanical systems with contact, impact and friction
Simulation of the ExoMars Rover (INRIA/Trasys Space/ESA)
State-of-the-art

Numerical time-integration methods for Nonsmooth Multibody systems (NSMBS):

Nonsmooth event capturing methods (Time-stepping methods)

- robust, stable and proof of convergence
- low kinematic level for the constraints
- able to deal with finite accumulation
- very low order of accuracy even in free flight motions

Two main implementations

- Moreau–Jean time-stepping scheme
- Schatzman–Paoli time-stepping scheme
Moreau’s Time stepping scheme [??]

Principle

\[
\begin{align*}
M(q_{k+\theta})(v_{k+1} - v_k) - hF_{k+\theta} &= p_{k+1} = G(q_{k+\theta})P_{k+1}, \\
q_{k+1} &= q_k + hv_{k+\theta}, \\
U_{k+1} &= G^T(q_{k+\theta})v_{k+1} \\
0 &\leq U_{k+1}^\alpha + eU_k^\alpha \perp P_{k+1}^\alpha \geq 0 \quad \text{if} \quad \bar{g}_k^\alpha,\gamma \leq 0 \\
P_{k+1}^\alpha &= 0 \quad \text{otherwise}
\end{align*}
\]

with

- $\theta \in [0, 1]$
- $x_{k+\theta} = (1 - \theta)x_{k+1} + \theta x_k$
- $F_{k+\theta} = F(t_k\theta, q_{k+\theta}, v_{k+\theta})$
- $\bar{g}_k,\gamma = g_k + \gamma hU_k, \gamma \geq 0$ is a prediction of the constraints.
Schatzman's Time stepping scheme

Principle

\[
\begin{aligned}
& M(q_k + 1)(q_{k+1} - 2q_k + q_{k-1}) - h^2 F_{k+\theta} = p_{k+1}, \\
& v_{k+1} = \frac{q_{k+1} - q_{k-1}}{2h}, \\
& -p_{k+1} \in N_K \left( \frac{q_{k+1} + eq_{k-1}}{1 + e} \right),
\end{aligned}
\]

where \( N_K \) defined the normal cone to \( K \).

For \( K = \{ q \in \mathbb{R}^n, y = g(q) \geq 0 \} \)

\[
0 \leq g \left( \frac{q_{k+1} + eq_{k-1}}{1 + e} \right) \perp \nabla g \left( \frac{q_{k+1} + eq_{k-1}}{1 + e} \right) P_{k+1} \geq 0
\]
Comparison

Shared mathematical properties

- Convergence results for one constraints
- Convergence results for multiple constraints problems with acute kinetic angles
- No theoretical proof of order

Mechanical properties

- Position vs. velocity constraints
- Respect of the impact law in one step (Moreau) vs. Two-steps (Schatzman)
- Linearized constraints rather than nonlinear.

But

These schemes do not satisfy both the position and velocity constraints in discrete time.
Objectives & means

Objectives
Design nonsmooth event capturing (time-stepping) methods with

- Same properties as standard methods (robustness, accumulation, ...)
- Controlled drift of the constraints also for the perfect bilateral constraints
- Study of multi-body systems with clearances. (Influence of clearance on stability, avoiding violations of the same of order of clearances)
- Consistency with CAD tool and detection collision engines (which suffer even from slight violations)

Means
- Projection algorithms onto the manifold defined by the constraints
Illustrations

Pendulum in a ring

Figure: Pendulum in a ring